HyperScience International Journal
Original Research Papers
Open Access Journals
ISSN: 2821-3300
HIJ, Vol 1, No 1, pp 1-7, Dec 2021
https://doi.org/10.55672/hij2021pp1-7

Generalization of Vaidya's Metric for A Radiating Star to Rotating Star

J. J. Rawal [1]
Bijan Nikouravan [1, 2*]

[1] *The Indian Planetary Society (IPS), B-201, Vishnu Apartment, Lokmanya Tilak Road, Borivali (West), Mumbai – 400092 (India),*
[2] *Department of Physics, Islamic Azad University (IAU)-Varamin-Pishva Branch, Iran*

ABSTRACT
Schwarzschild external solution of Einstein's gravitational field equations in general theory of relativity for a static star has been generalised by Vaidya [1], taking into account the radiation of the star. Here, we generalise Vaidya's metric to a star which is rotating and radiating. Although, there is famous Kerr solution [2] for a rotating star, but here is a simple solution for a rotating star which may be termed as zero approximate version of Kerr solution. Results are discussed.

Keyword: Vaidya metric for a radiating star-its generalization to a slowly rotating star.

INTRODUCTION

Schwarzschild's external solution deals with the gravitational field of a cold dark body whose mass is constant. The application of this solution to describe the Sun's gravitational field should only be regarded as approximate. Vaidya [1], generalized Schwarzschild's static solution for radiating star. However, this is also not fully generalized solution for a radiating star as the star rotates while it is radiating. Here, we generalized Vaidya's metric for a radiating star to rotating star. Results are discussed.

SCHWARZSCHILD-LIKE SOLUTION FOR A ROTATING STAR

The line element in Cartesian four-dimensional free space with time as its fourth dimension is given by

$$ds^2 = -dx^2 - dy^2 - dz^2 + c^2dt^2 \tag{1}$$

$$ds^2 = -dx^2 - dy^2 - dz^2 + dt^2 \tag{2}$$

Where c = 1. We transform it to a rotating coordinate system with the transformation equations

$$\begin{gathered} x = x'Cos\theta - y'Sin\theta \\ y = x'Sin\theta + y'Cos\theta \,, \\ z = z', \quad t = t', \quad \omega = \frac{\theta}{t} \end{gathered} \tag{3}$$

that is, $\theta = \omega t$ and ω is a constant angular velocity of a star or any celestial body which is, of course, less than its critical rotational velocity.
We have

$$\begin{aligned} ds^2 = -dx'^2 - dy'^2 - dz'^2 + [1 - \omega^2 r^2]dt^2 \\ + 2\omega t(x'dy' - y'dx') \end{aligned} \tag{4}$$

Dropping the suffixes, we have

$$ds^2 = -dx^2 - dy^2 - dz^2 + [1-\omega^2 r^2]dt^2 + 2\omega t(xdy - ydx) \tag{5}$$

Transforming this into spherical polar coordinates

$$x = rsin\theta cos\varphi, \quad y = rsin\theta sin\varphi, \quad z = rcos\theta \tag{6}$$

We have

$$ds^2 = -dr^2 - r^2 d\theta^2 - r^2 sin^2\theta d\varphi^2 + 2\omega r^2 sin^2\theta d\varphi\, dt + [1 - r^2\omega^2 sin^2\theta]dt^2 \tag{7}$$

In the presence of matter, the metric Eq (7), takes the form

$$ds^2 = -e^\lambda dr^2 - r^2 d\theta^2 - r^2 sin^2\theta d\varphi^2 + 2\omega r^2 sin^2\theta\, d\varphi\, dt + e^\nu[1 - r^2\omega^2 sin^2\theta]dt^2$$

Where

$$\lambda = \lambda(r) \quad \text{and} \quad \nu = \nu(r) \tag{8}$$

Sixteen components of covariant metric tensor $[g_{ij}]$ are given by

$$[g_{ij}] = \begin{bmatrix} -e^\lambda & 0 & 0 & 0 \\ 0 & -r^2 & 0 & 0 \\ 0 & 0 & -r^2\sin^2\theta & \omega r^2\sin^2\theta \\ 0 & 0 & \omega r^2\sin^2\theta & e^\nu[1 - r^2\omega^2\sin^2\theta] \end{bmatrix} \tag{9}$$

The determinant $g = |g_{ij}|$ is given by

$$g = -e^{\lambda+\nu} r^4 sin^2\theta \tag{10}$$

and, therefore, sixteen components of contravariant metric tensors $[g^{ij}]$ are given by

$$[g^{ij}] = \begin{bmatrix} -e^{-\lambda} & 0 & 0 & 0 \\ 0 & -1/r^2 & 0 & 0 \\ 0 & 0 & \dfrac{-[1-\omega^2 r^2\sin^2\theta]}{r^2\sin^2\theta} & \omega e^{-\nu} \\ 0 & 0 & \omega e^{-\nu} & e^{-\nu} \end{bmatrix} \tag{11}$$

Sixty-four Christoffel Symbols of the Second kind are given by the formula

$$\Gamma^i_{jk} = \frac{1}{2} g^{il}\left[\frac{\partial_{lj}}{\partial x^k} + \frac{\partial g_{lk}}{\partial x^j} - \frac{\partial g_{jk}}{\partial x^l}\right] \tag{12}$$

Out of these 64 Christoffel Symbols of the Second kind, the following 21 are non-vanishing and the rest of them are vanishing.

$$\Gamma^1_{11} = \frac{\lambda'}{2}, \quad \Gamma^1_{22} = -e^{-\lambda} r, \quad \Gamma^1_{34} = r\omega e^{-\lambda} sin^2\theta$$

$$\Gamma^1_{33} = -re^{-\lambda} sin^2\theta, \quad \Gamma^1_{44} = e^{\nu-\lambda}\frac{\nu'}{2} - r\omega^2 sin^2\theta$$

$$\Gamma^2_{12} = {}^1/_r, \quad \Gamma^2_{33} = -sin\theta\, cos\theta$$

$$\Gamma^2_{34} = \omega sin\theta\, cos\theta, \quad \Gamma^2_{44} = -\omega^2 sin\theta cos\theta$$

$$\Gamma^3_{13} = \frac{1}{r}, \quad \Gamma^3_{14} = -\frac{\omega}{r} + \frac{\omega\nu'}{2}$$

$$\Gamma^3_{23} = cot\theta, \quad \Gamma^3_{24} = -\omega cot\theta, \quad \Gamma^4_{14} = {}^{\nu'}/_2 \tag{13}$$

Here dash (′) denotes the differentiation ω. r. t. r. Einstein's gravitational field equations in free space are given by

$$G_{ij} = 0 \tag{14}$$

Where

$$G_{ij} \equiv \frac{\partial^2}{dx^i \partial x^j}[log\sqrt{-g}] - \frac{\partial}{\partial x^k}\Gamma^k - \Gamma^k_{ij}\frac{\partial}{dx^k}[log\sqrt{-g}] + \Gamma^k_{jl}\Gamma^l k_i \tag{15}$$

Eqns. (14) are sixteen equations out of which five are identically not equal to zero, rest of eleven are identically equal to zero. These five eqns. are the following,

$$G_{11} \equiv \frac{\nu''}{2} - \frac{\lambda'\nu'}{4} - \frac{\lambda'}{r} + \frac{\nu'^2}{4} = 0 \tag{16a}$$

$$G_{22} \equiv e^{-\lambda}\left[1-\frac{r\lambda'}{2}+\frac{r\nu'}{2}\right]-1=0 \tag{16b}$$

$$G_{33} \equiv sin^2\theta\left[e^{-\lambda}\left\{1-\frac{r\lambda'}{2}+\frac{r\nu'}{2}\right\}-1\right]=0 \tag{16c}$$

or

$$G_{33} \equiv \left[e^{-\lambda}\left\{1-\frac{r\lambda'}{2}+\frac{r\nu'}{2}\right\}-1\right]=0$$

$$G_{44} \equiv \left[\frac{\lambda'\nu'}{4}-\frac{\nu'^2}{4}-\frac{\nu''}{2}-\frac{\nu'}{r}\right]=0 \tag{16d}$$

or

$$G_{44} \equiv e^{\nu-\lambda}\left[\frac{\lambda'\nu'}{4}-\frac{\nu'^2}{4}-\frac{\nu''}{2}-\frac{\nu'}{r}\right]=0$$

$$G_{34} \equiv -\omega sin^2\theta\left[e^{-\lambda}\left\{1-\frac{r\lambda'}{2}+\frac{r\nu'}{2}\right\}-1\right]=0 \tag{16e}$$

or

$$G_{34} \equiv \left[e^{-\lambda}\left\{1-\frac{r\lambda'}{2}+\frac{r\nu'}{2}\right\}-1\right]=0$$

In Eqns. (16e), the angular velocity ω appears, showing the rotation of the star. Eqn. (16e) does not appear in Static Schwarzschild equations. This is additional equation due to rotation of the star. Solving these equations, we get

$$ds^2 = -\left[1-\frac{2\{m+(1/\omega)\}}{r}\right]^{-1} dr^2 - r^2 d\theta^2 - r^2 sin^2\theta\, d\phi^2 + 2\omega r^2 sin^2\theta d\phi dt + \left\{[1-r^2\omega^2 sin^2\theta]\left[1-\frac{2\{m+(1/\omega)\}}{r}\right]\right\} dt^2 \tag{17}$$

Note that m and (1 / ω) both have dimensions of length.

$$\left[1-\frac{2\{m+(1/\omega)\}}{r}\right]=0 \tag{18}$$

represents the singularity of the solution.
Here $r = 2\,(m+1/\omega)$ represents event horizon distance. It is interesting to note that, here, we get a solution for empty universe, when we take $m = 0$. In this case the solution is

$$ds^2 = -\left[1-\frac{2/\omega}{r}\right]^{-1} dr^2 - r^2 d\theta^2 - r^2 sin^2\theta d\phi^2 + 2\omega r^2 sin^2\theta\, d\phi\, dt + [1-r^2\omega^2 sin^2\theta]\left[1-\frac{2/\omega}{r}\right] dt^2 \tag{19}$$

This solution has singularity at a distance given by $r = (2/\omega)$. This shows that the angular velocity, that is, the rotation of the star changes the gravitational field. It increases the gravitational field in more amount in anti-clockwise direction, that is, when $\omega > 0$; as ω is small, rotation's interaction time with space is large, and it increases but in small amount if ω is large, as rotation's interaction time with space is short, and opposite is true when $\omega < 0$.
Rotation of a star affects the shape and structure of the star, depending upon the value of ω. ω is always less than the critical rotational velocity. If we take value of ω more than star's critical rotational velocity, star breaks down into pieces. ω of a star has its own critical velocity ω_c as its upper bound. Work presented here may be termed as zero approximate version of Kerr solution [2].

ENERGY TENSOR FOR A DIRECTED FLOW OF RADIATION

Following Vaidya [3-5], Narlikar [6, 7] , [8], [9] [4] by the term "directed flow of radiation" it is meant a distribution of electro-magnetic energy such that a local observer at any point of the region of space under consideration finds one and only one direction in which the radiant energy is flowing at the point. Using the natural co-ordinates at that point of interest, we may take the components of energy tensor as being given in terms of electric and magnetic field strengths $\vec{E}$ and $\vec{B}$ by the typical example given by [10].

$$T_0^{11} = -\tfrac{1}{2}\left(E_x^2 - E_y^2 - E_z^2 + H_x^2 - H_y^2 - H_z^2\right) \tag{20.a}$$

$$T_0^{12} = -\left(E_x E_y + H_x H_y\right) \tag{20.b}$$

$$T_0^{14} = \left(E_y H_z - E_z H_y\right) \tag{20.c}$$

$$T_0^{44} = -\tfrac{1}{2}\left(E_x^2 + E_y^2 + E_z^2 + H_x^2 + H_y^2 + H_z^2\right) \tag{20.d}$$

The suffix 0 to a component of a tensor indicates that the component is evaluated in natural coordinates at the point of interest. Considering for simplicity that the axes of our natural coordinates are oriented in such a way that the flow of radiation at the point of interest is in the X-direction and further that the radiation is polarized with the electric vector parallel to the Y-direction. Therefore, we have,

$$E_x = E_z = H_x = H_y = 0 \quad \text{and} \quad E_y = H_z \tag{21}$$

Therefore, the only surviving components of the tensor $T_0^{\mu\nu}$ would be,

$$T_0^{11} = T_0^{44} = T_0^{14} = \frac{1}{2}\left(E_x^2 + E_y^2\right) = \rho \tag{22}$$

ρ being the density of the radiant energy at the point. Having obtained the components of $T_0^{\mu\nu}$ for one system of coordinates, we can find them in every other system by the rules of tensor transformation. For a general coordinate system with a line element

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \tag{23}$$

The Components of $T_0^{\mu\nu}$ will be given by

$$T^{\mu\nu} = \frac{\partial x^\mu}{\partial x_0^\alpha}\frac{\partial x^\nu}{\partial x_0^\beta} T_0^{\alpha\beta} \tag{24}$$

Using (22), these yields,

$$T^{\mu\nu} = \left[\frac{\partial x^\mu}{\partial x_0^1}\frac{\partial x^\nu}{\partial x_0^1} + \frac{\partial x^\mu}{\partial x_0^4}\frac{\partial x^\nu}{\partial x_0^4} + \frac{\partial x^\mu}{\partial x_0^1}\frac{\partial x^\nu}{\partial x_0^4} + \frac{\partial x^\mu}{\partial x_0^4}\frac{\partial x^\nu}{\partial x_0^1}\right]\rho \tag{25}$$

As the radiant energy travels along null-geodesics

$$dx_0^1 = dx_0^4 = d\tau \tag{26}$$

By (25) and (26), along with radiation flow, we find

$$g_{\mu\nu}\, dx^\mu\, dx^\nu = 0 \tag{27}$$

We use (26) in

$$\frac{dx^\mu}{d\tau} = \frac{\partial x^\mu}{\partial x_0^\alpha}\frac{dx_0^\alpha}{d\tau}$$

and find,

$$\frac{dx^\mu}{d\tau} = \frac{\partial x^\mu}{\partial x_0^1} + \frac{\partial x^\mu}{\partial x_0^4} \tag{28}$$

With the help of (28), (25) reduces to

$$T^{\mu\nu} = \rho \frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} \tag{30}$$

with

$$g_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} = 0 \tag{31}$$

Thus, for the case of the outside field of radiating star the energy tensor is to be taken of the form

$$T^{\mu\nu} = \rho v^\mu v^\nu \tag{32}$$

With

$$v_\mu v^\nu = 0 \quad ; \quad (v^\mu)_\nu v^\nu = 0 \tag{33}$$

THE FIELD EQUATIONS

A star of mass M and radius r_0 rotating with a constant angular velocity ω, which is less than its critical velocity, is supposed to start radiating at time t_0. As the star continues to radiate the zone of radiation increases in thickness, its outer surface at a later instant t_1 being $r = r_1$. For $r_0 \le r \le r_1$, $t_0 \le t \le t_1$. We are considering the following metric,

$$ds^2 = -e^\lambda dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2 + 2\omega r^2 \sin^2\theta\, d\phi\, dt + e^\nu dt^2 \tag{33}$$

Where

$$e^\lambda = \left[1 - \frac{2\left(m + \left(\frac{1}{\omega}\right)\right)}{r}\right]^{-1} \tag{33a}$$

and

$$e^\nu = \left\{\left[1 - r^2\omega^2 \sin^2\theta\right]\left[1 - \frac{2\left(m + \left(\frac{1}{\omega}\right)\right)}{r}\right]\right\} \tag{33b}$$

Here, we have $\lambda = \lambda(r,t)$ and $\nu = \nu(r,t)$. For the nature of radiation, Vaidya has found the energy Tensor $T^{\mu\nu}$ is of the form

$$T^{\mu\nu} = \rho v^{\mu} v^{\nu} \tag{34}$$

so that ρ is the density of radiation and the lines of flow are null-geodesic

$$v_{\mu} v^{\nu} = 0 \quad ; \quad (v^{\mu})_{\nu} v^{\nu} = 0 \tag{35}$$

Since $(T^{\mu\nu})_{\nu} = 0$, we have the analogue of the equation of continuity

$$(\rho v^{\mu})_{\mu} = 0 \tag{36}$$

As the flow is to be radial, $v^2 = 0, v^3 = 0$ and

$$T_1^1 = \rho v_1 v^1, \quad T_4^4 = \rho v_4 v^4,$$

$$T_1^4 = \rho v_1 v^4, \quad T_2^2 = T_3^3 = 0 \tag{37}$$

Because $\lambda = \lambda(r,t)$ and $v = v(r,t)$ here, the additional Γ's, other than (13), are

$$\Gamma_{14}^1 = \frac{\dot{\lambda}}{2}, \qquad \Gamma_{11}^3 = \frac{\omega\lambda}{2},$$
$$\Gamma^3_{44} = \frac{\omega v'}{2}, \qquad \Gamma_{44}^4 = \frac{v'}{2} \tag{38}$$

Also $v_{\mu} v^{\nu} = 0$ simplifies to

$$-e^{\lambda}(v^1)^2 + e^{v}(v^4)^2 = 0 \tag{39}$$

With the usual expression for the components of $T^{\mu\nu}$ in terms of $g_{\mu\nu}$ and their derivatives, (37) gives the following three equations

(i)

$$T_1^4 e^{(v-\lambda)/2} + T_4^4 = 0 \tag{40}$$

or

$$e^{-\lambda}\left(\frac{\lambda'}{r} - \frac{1}{r^2}\right) + \frac{1}{r^2} + \frac{\dot{\lambda}}{r} e^{-(v+\lambda)/2} = 0 \tag{41}$$

(ii)

$$T_1^1 + T_4^4 = 0 \tag{42}$$

or

$$e^{-\lambda}\left(\frac{\lambda'}{r} - \frac{v'}{r^2} - \frac{2}{r^2}\right) + \frac{2}{r^2} = 0 \tag{43}$$

(iii)

$$T_2^2 = 0 \tag{44}$$

or

$$-e^{-\lambda}\left[\frac{v''}{2} + \frac{v'^2}{4} - \frac{\lambda' v'}{4} + \frac{v'}{2r} - \frac{\lambda'}{2r}\right] + e^{-v}\left[\frac{\ddot{\lambda}}{2} + \frac{\dot{\lambda}^2}{4} - \frac{\dot{\lambda}\dot{v}}{4}\right] = 0 \tag{45}$$

Here dash (') denotes the differentiation with respect to r and dot (.) that with respect to time. If the total energy is to be conserved, the line-element obtained by solving above mentioned equations must reduce to the line element of the rotating star with mass M given by,

$$ds^2 = -\left[1 - \frac{2\left(M + \frac{1}{\omega}\right)}{r}\right] dr^2 - r^2 d\theta^2 - r^2 sin^2\theta d\phi^2 + 2\omega r^2 sin^2\theta\, d\phi\, dt + \left\{[1 - r^2\omega^2 sin^2\theta\,]\left[1 - \frac{\left(M + \frac{1}{\omega}\right)}{r}\right]\right\} dt^2 \tag{46}$$

Where M is the constant mass of the star which does not depend either on r or t at $r = r_0$, $t = t_0$ and for $r \geq r_1$ at $t = t_1$.

THE SOLUTION OF FIELD EQUATIONS

On putting

$$e^{-\lambda} = \left[1 - \frac{2\left(m + \frac{1}{\omega}\right)}{r}\right] \tag{47}$$

or

$$e^{-\lambda} = \left[1 - \frac{2\bar{m}}{r}\right]$$

where

$$\bar{m} = \left(m + \frac{1}{\omega}\right) \tag{48}$$

As ω (the angular velocity is constant)

$$\dot{\bar{m}} = \dot{m} \quad \text{and} \quad \bar{m}' = m' \tag{49}$$

In the field of equation (41), we find that it is equivalent to

$$e^{-\lambda/2}\frac{\partial\bar{m}}{\partial r} - e^{-v/2}\frac{\partial\bar{m}}{\partial t} = 0 \tag{50}$$

Using the operator

$$\frac{d}{d\tau} \equiv \nu^1 \frac{\partial}{\partial r} + \nu^4 \frac{\partial}{\partial t} \tag{51}$$

Which may be expressed as

$$\frac{d\bar{m}}{d\tau} = 0 \tag{52}$$

From (51), we can express $e^{\nu/2}$ in terms of M

$$e^{\nu/2} = \frac{\dot{m}}{\bar{m}'}\left[1 - \frac{2\bar{m}}{r}\right]^{-\frac{1}{2}} \tag{53}$$

Now we can take the second field equation (43). On substitution, the values of λ and ν from (48) and (50), we find that

$$\left(\frac{\dot{m}'}{\dot{m}} - \frac{m''}{m'}\right)\left(1 - \frac{2\bar{m}}{r}\right) = \frac{2\bar{m}}{r^2} \tag{54}$$

One can verify that the first integral of the above equation is

$$m'\left(1 - \frac{2\bar{m}}{r}\right) = f(\bar{m}) \tag{55}$$

$f(\bar{m})$ being an arbitrary function of $\bar{m}$. Taking (45), we shall show that when λ *and* ν are given by (48), (53), together with (55) the equation (44) is automatically satisfied.
The following is an identity holding between the components of the tensor T_μ^ν

$$\frac{\partial}{\partial r}(T_1^1) + \frac{\partial}{\partial t}(T_1^4) - \frac{\nu'}{2}(T_4^4 - T_1^1) + \frac{2}{r}(T_1^1 - T_2^2) + T_1^4\left(\frac{\dot{\lambda} + \dot{\nu}}{2}\right) = 0 \tag{56}$$

With the help of this identity and two other equations (40) and (42), the equation (44) can be transformed into

$$\frac{d}{d\tau}\left(r^2 e^{-\lambda} T_4^4\right) = 0 \tag{57}$$

Thus, the third field equation is satisfied. i.e : $T_2^2 = 0$, provided (57) is satisfied i.e. provided

$$\frac{d}{d\tau}\left\{m'\left(1 - \frac{2\bar{m}}{r}\right)\right\} = 0 \tag{58}$$

i.e provided $\frac{d\bar{m}}{d\tau} = 0$, when we use (51). And the last relation is already proved as (52) above. Hence, we have solved all the field equations and the final line element describing the radiation envelope of a star is

$$ds^2 = -\left[1 - \frac{2\left(m + \frac{1}{\omega}\right)}{r}\right]^{-1} dr^2 - r^2 d\theta^2 - r^2 sin^2\theta d\phi^2 + 2\omega r^2 sin^2\theta\, d\phi\, dt + [1 - r^2\omega^2 sin^2\theta]\frac{\dot{m}^2}{f^2}\left[1 - \frac{2\left(m + \frac{1}{\omega}\right)}{r}\right] dt^2$$

With m'

$$\left[1 - \frac{2\left(m + \frac{1}{\omega}\right)}{r}\right] = f\left\{m + \frac{1}{\omega}\right\} \tag{59}$$

$\bar{m} = \bar{m}$ (r, t). For $r_0 \leq r \leq r_1$, $t_0 \leq t \leq t_1$.
The surviving components of the energy tensor are,

$$-T_1^1 = T_4^4 = \frac{m'}{4\pi r^2}\ ,\quad T_1^4 = \frac{m'^2}{4\pi\dot{m}r^2}\ ,\quad T_4^1 = \frac{-\dot{m}}{4\pi r^2} \tag{60}$$

With (60), we can even write the line element as

$$ds^2 = -\left[1 - 2\frac{\{m + (1/\omega)\}}{r}\right]^{-1} dr^2 - r^2 d\theta^2 - r^2 sin^2\theta d\phi^2 + 2\omega r^2 sin^2\theta d\phi dt + \frac{[1 - r^2\omega^2 sin^2\theta]\dot{m}^2\left[1 - 2\frac{\{m + 1/\omega\}}{r}\right] dt^2}{\left[m'\left\{1 - 2\frac{(m + 1/\omega)}{r}\right\}\right]^2} \tag{61}$$

$$= -\left[1 - 2\frac{\{m + (1/\omega)\}}{r}\right]^{-1} dr^2 - r^2 d\theta^2 - r^2 sin^2\theta d\phi^2 + 2\omega r^2 sin^2\theta d\phi dt + [1 - r^2\omega^2 sin^2\theta]\frac{\dot{m}^2}{m'^2}\left[1 - 2\frac{\{m + (1/\omega)\}}{r}\right]^{-1} dt^2 \tag{62}$$

$$= -\left[1 - 2\frac{\{m + (1/\omega)\}}{r}\right]^{-1} dr^2 - r^2 d\theta^2 - r^2 sin^2\theta d\phi^2 + 2\omega r^2 sin^2\theta d\phi dt + [1 - r^2\omega^2 sin^2\theta]\frac{\dot{m}^2}{m'^2} e^\lambda dt^2 \quad (63)$$

where

$$e^\lambda = \left[1 - 2\frac{\{m + (1/\omega)\}}{r}\right]^{-1} \quad (64)$$

or that,

$$ds^2 = -e^\lambda dr^2 - r^2 d\theta^2 - r^2 sin^2\theta d\phi^2 + 2\omega r^2 sin^2\theta d\phi dt + [1 - r^2\omega^2 sin^2\theta]\left(\frac{\dot{m}}{m'}\right)^2 e^\lambda dt^2 \quad (66)$$

where

$$e^\lambda = \left[1 - \frac{2\left(m + \frac{1}{\omega}\right)}{r}\right]^{-1} \quad (67)$$

This is the metric (gravitational field) of a rotating star with envelope of radiation. In this appears angular velocity ω, as well as the variation of mass m in the form of $\dot{m}$ and m' that is with respect to time and along the radial vector. This difference in the original mass is converted into radiation. At $r = r_0$, t = t_0 $and\ r = r_1$, $t = t_1$, the metric describing the field of the rotating star with the angular velocity ω of which the mass is constant M.
In the interval $r_0 \leq r \leq r_1$, $t_0 \leq t \leq t_1$, the line element describes its gravitational field of a radiating star in the general theory of relativity. This is generalisation of Vaidya's metric for radiating star to the rotating star. It is to be noted that rotation of the star does not contribute anything in the process of generating radiation.
Following [5], it is easy to see that m, v^1 $and\ r^2\rho$ are conserved along the lines of flow, and that $f(\overline{m}) \simeq m' or - \dot{m}$ because m' ≃ -$\dot{m}$ showing that $f(\overline{m})$ measures the luminosity of the star at the Newtonion level of approximation. Following [5] it is also verified that the principle of conservation of energy holds good.

REFERENCES

[1] P. C. Vaidya, "The gravitational field of a radiating star," in *Proceedings of the Indian Academy of Sciences-Section A*, 1951, vol. 33, no. 5, p. 264: Springer.
[2] R. P. J. P. r. l. Kerr, "Gravitational field of a spinning mass as an example of algebraically special metrics," vol. 11, no. 5, p. 237, 1963.
[3] P. J. c. s. Chunilal Vaidya, "The external field of a radiating star in general relativity," vol. 12, p. 183, 1943.
[4] V. Narlikar and P. J. N. Vaidya, "A Spherically Symmetrical Non-Static Electromagnetic Field," vol. 159, no. 4045, pp. 642-642, 1947.
[5] P. J. N. Vaidya, "A Radiation-absorbing Centre in a Non-statical Homogeneous Universe," vol. 166, no. 4222, pp. 565-565, 1950.
[6] V. J. M. N. o. t. R. A. S. Narlikar, "The stability of a particle in a gravitational filed," vol. 96, p. 263, 1936.
[7] V. J. T. L. Narlikar, Edinburgh,, D. P. Magazine, and J. o. Science, "II. The concept and determination of mass in Newtonian mechanics," vol. 27, no. 180, pp. 33-36, 1939.
[8] V. Narlikar, D. J. T. L. Moghe, Edinburgh,, D. P. Magazine, and J. o. Science, "LXXXVII. Some new solutions of the differential equation for isotropy," vol. 20, no. 137, pp. 1104-1108, 1936.
[9] V. Narlikar and P. J. P. N. I. S. Vaidya, "Non-static electromagnetic fields with spherical symmetry," vol. 14, p. 53, 1948.
[10] A. J. S. Einstein, "Relativity, thermodynamics and cosmology," vol. 80, no. 2077, pp. 358-358, 1934.