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Mathematical-Physical Approach to Prove that the Navier-Stokes Equations Provide a Correct Description of Fluid Dynamics

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ABSTRACT

This publication takes a mathematical approach to a general solution to the Navier-Stokes equations. The basic idea is a mathematical analysis of the unipolar induction according to Faraday with the help of the vector analysis. The vector analysis enables the unipolar induction and the Navier-Stokes equations to be related physically and mathematically, since both formulations are mathematically equivalent. Since the unipolar induction has proven itself in practice, it can be used as a reference for describing the Navier-Stokes equations.

Keywords: Navier-Stokes equations; Fluid dynamics.

INTRODUCTION

The Navier-Stokes equations describe the movement of liquids and gases. The first problem with the set of equations is that the proof for a solution in three-dimensional space has not yet been produced. The second problem is that the math behind the equations is difficult to understand and has not yet been explained plausibly. The third problem is one of the so-called Millennium Prize problems and is to prove the generality of the equations [1]. This paper deals with the third problem and at the same time solves the first two problems. Vector calculation was not yet introduced during the lifetime of Claude Louis Marie Henri Navier (1785-1836) and was still in its infancy during the lifetime of George Gabriel Stokes (1819-1903) (it was introduced in 1844) [2]. In this paper a proposal is formulated with which the Navier-Stokes equations can be derived from vector calculations in order to solve the three problems listed above. A mathematical connection to the unipolar induction according to Faraday and thus to the "Maxwell equations" [3] is established in order to prove that the Navier-Stokes equations [4] are also valid in three-dimensional space. To explain the approach, the Navier-Stokes equations for incompressible Newtonian liquids at constant pressure are combined with the equations for the unipolar induction according to Faraday, through vector analysis. The aim of this thesis is not to explain the already known and recognized mathematical principles of the vector calculation. Reference is only made to this to explain the approach.

IDEAS AND METHODS: IDEA BEHIND THE SOLUTION

The idea is to apply the vector description of the unipolar induction to the Navier-Stokes equations. This explains a general validity for physical behaviour with regard to the movement of substances of all kinds and the effect of forces on these substances. Unipolar induction [5, 6].

$$\vec{E} = \vec{v} \times \vec{B} \tag{1}$$

$\vec{E}$ = electric field strength, $\vec{v}$ = velocity, $\vec{B}$ = magnetic flux density, μ = magnetic permeability, $\vec{H}$ = magnetic field strength. If the rot-operator (Curl operator) is now used on both sides of equations (1) and (2) results.

$$\vec{\nabla} \times \vec{E} = \vec{\nabla} \times (\vec{v} \times \vec{B}) \tag{2}$$

According to the rules of vector analysis, equation (2) can also be rewritten as equation (3).

$$\vec{\nabla} \times \vec{E} = \vec{\nabla} \times (\vec{v} \times \vec{B}) = (\vec{B} \cdot \vec{\nabla})\, \vec{v} - (\vec{v} \cdot \vec{\nabla})\, \vec{B} + \vec{v}\, (\vec{\nabla} \cdot \vec{B}) - \vec{B}\, (\vec{\nabla} \cdot \vec{v}) \tag{3}$$

The key message of this formula is that a magnetic field is created when an object moves through an electric field. The material constant μis given by the relationship $\vec{B} = \mu\vec{H}$. If $\vec{E} = \overrightarrow{\Phi_1}$, $\vec{H} = \overrightarrow{\Phi_2}$ and $\mu = a$ are now abstracted, the equation (4) arises.

$$\vec{\nabla} \times \overrightarrow{\Phi_1} = \vec{\nabla} \times \left(\vec{v} \times (a\overrightarrow{\Phi_2})\right) = \left((a\overrightarrow{\Phi_2}) \cdot \vec{\nabla}\right) \vec{v} - (\vec{v} \cdot \vec{\nabla})(a\overrightarrow{\Phi_2}) + \vec{v}\, (\vec{\nabla} \cdot (a\overrightarrow{\Phi_2})) - (a\overrightarrow{\Phi_2})(\vec{\nabla} \cdot \vec{v}) \tag{4}$$

If the terms of equation (4) are now mathematically reformulated, a new overall expression is created which has an analogy to the Navier-Stokes equations. This expression is shown in equation (5).

$$\vec{\nabla} \times \overrightarrow{\Phi_1} = a\, (\overrightarrow{\Phi_2} \cdot \vec{\nabla})\, \vec{v} - a\, \frac{\delta\overrightarrow{\Phi_2}}{\delta t} + \vec{v}\, (\vec{\nabla} \cdot (a\overrightarrow{\Phi_2})) - (a\overrightarrow{\Phi_2})(\vec{\nabla} \cdot \vec{v}) \tag{5}$$

If the equation 5 is now multiplied by -1, the result is equation (6).

$$-(\vec{\nabla} \times \overrightarrow{\Phi_1}) = -a\, (\overrightarrow{\Phi_2} \cdot \vec{\nabla})\, \vec{v} + a\frac{\delta\overrightarrow{\Phi_2}}{\delta t} - \vec{v}\, (\vec{\nabla} \cdot (a\overrightarrow{\Phi_2})) + (a\overrightarrow{\Phi_2})(\vec{\nabla} \cdot \vec{v}) \tag{6}$$

In direct comparison, the equations (7) and (8) are the Navier-Stokes equations [4].

$$f = \rho\frac{\delta u}{\delta t} + \rho(u \cdot \vec{\nabla})\, u - (\vec{\nabla} \cdot \sigma_{(u,p)}) + 0 \tag{7}$$

and

$$(\vec{\nabla} \cdot u) = 0 \tag{8}$$

Here and also in the following explanations $u$is equated with the expression of the velocity $\vec{v}$. Since Φ_2 must be based on a field which contains sources and sinks, i.e. in which density distributions play a role, and which occurs in n-dimensional space, we can assume that the Navier-Stokes equations also have the effect in map n-dimensional space. The reason for this is that the "Maxwell-Equations", which can also be derived from the unipolar induction, have proven to be a consistent description of electromagnetic fields to this day.

BASICS OF VECTOR CALCULATION

In order to be able to derive the set of equations of the Navier-Stokes equations from vector calculation, this chapter describes the fundamentals of vector calculation used to solve the problems described in chapter 1 introduction of this paper [5].

First of all, three meta vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are introduced at this point. These three meta vectors are used in the basic mathematical description of the cross product in equation 9.

$$\vec{c} = \vec{a} \times \vec{b} \tag{9}$$

In equation (10), the rot-operator is used on both sides of equation (9).

$$\vec{\nabla} \times \vec{c} = \vec{\nabla} \times (\vec{a} \times \vec{b}) \tag{10}$$

Now the right side of equation (10) is rewritten according to the calculation rules of vector calculation and equation (11) arises.

$$\vec{\nabla} \times \vec{c} = \vec{\nabla} \times (\vec{a} \times \vec{b}) = (\vec{b} \cdot \vec{\nabla})\, \vec{a} - (\vec{a} \cdot \vec{\nabla})\, \vec{b} + \vec{a}\, (\vec{\nabla} \cdot \vec{b}) - \vec{b}(\vec{\nabla} \cdot \vec{a}) \tag{11}$$

When equation (11) is multiplied by -1, the expression results from equation (12).

$$-(\vec{\nabla} \times \vec{c}) = -(\vec{b} \cdot \vec{\nabla})\, \vec{a} + (\vec{a} \cdot \vec{\nabla})\, \vec{b} - \vec{a}\, (\vec{\nabla} \cdot \vec{b}) + \vec{b}\, (\vec{\nabla} \cdot \vec{a}) \tag{12}$$

SUBSTITUTING THE PHYSICAL COMPONENTS OF THE NAVIER-STOKES EQUATIONS

In the next step, the meta vector $\vec{a}$ in equation 12 is replaced by the velocity vector $\vec{v}$. The meta vector $\vec{b}$ is replaced by the density multiplied by the velocity $(\rho \cdot \vec{v})$. The result is equation (13).

$$-\left((\rho\vec{v}) \cdot \vec{\nabla}\right)\vec{v} + (\vec{v} \cdot \vec{\nabla})(\rho\vec{v}) - \vec{v}\left(\vec{\nabla} \cdot (\rho\vec{v})\right) + (\rho\vec{v})(\vec{\nabla} \cdot \vec{v}) = -(\vec{\nabla} \times (\vec{v} \times (\rho\vec{v}))) \quad (13)$$

where $\vec{v}$ is velocity and ρ is density

NAVIER-STOKES EQUATIONS

The formulas of the Navier-Stokes equations and the vector calculation to which reference is made in this publication are presented here. Throughout the elaboration, the form of variation of the incompressible Navier-Stokes equations is referred to and used as a reference. The approach can also be used for other forms of variation of the Navier-Stokes equations, but then only with the application of the appropriate laws for the vector calculation [4]. Here $u = \vec{v}$ is velocity, t is time, ρ as density, σ is defined as Stress tensor, p as pressure and f is undefined force.

$$\rho\frac{\delta u}{\delta t} + \rho\left(u \cdot \vec{\nabla}\right)u - \left(\vec{\nabla} \cdot \sigma_{(u,p)}\right) = f \quad (14)$$

$$\vec{\nabla} \cdot u = 0 \quad (8)$$

$$\sigma_{(u,p)}n = h \quad (15)$$

The expression u is used here for the expression of the velocity $\vec{v}$. In order to get a better overview of the proposed solution, the equation (16, 17, 18) and (8) are written one above the other.

$$-\left(\vec{\nabla} \times (\vec{a} \times \vec{b})\right) = -(\vec{b} \cdot \vec{\nabla})\,\vec{a} + (\vec{a} \cdot \vec{\nabla})\,\vec{b} - \vec{a}\,(\vec{\nabla} \cdot \vec{b}) + \vec{b}\,(\vec{\nabla} \cdot \vec{a}) \quad (16)$$

$$-\left(\vec{\nabla} \times (\vec{v} \times (\rho\vec{v}))\right) = -\left((\rho\vec{v}) \cdot \vec{\nabla}\right)\vec{v} + (\vec{v} \cdot \vec{\nabla})(\rho\vec{v}) - \vec{v}\left(\vec{\nabla} \cdot (\rho\vec{v})\right) + (\rho\vec{v})(\vec{\nabla} \cdot \vec{v}) \quad (17)$$

$$f = \rho\left(u \cdot \vec{\nabla}\right)u + \rho\frac{\delta u}{\delta t} - \left(\vec{\nabla} \cdot \sigma_{(u,p)}\right) + 0 \quad (18)$$

with

$$\vec{\nabla} \cdot u = 0 \quad (8)$$

MATHEMATICAL APPROACH

In the following chapters, the mathematical-physical combination of the individual terms from equations (8), (17) and (18) is discussed in more detail. First of all, the second term in each case from equations (17) $\left((\rho\vec{v}) \cdot \vec{\nabla}\right)\vec{v}$ and (18) $\rho\left(u \cdot \vec{\nabla}\right)u$ is equated in equation (19). According to the commutative law of multiplication, the factor ρ can change its position as a factor. Therefore, it does not matter where the factor ρ is within both sides of equation (19).

$$\left((\rho\vec{v}) \cdot \vec{\nabla}\right)\vec{v} = \rho\left(u \cdot \vec{\nabla}\right)u \quad (19)$$

According to the rules of multiplication, the expression ρ from the right site of equation (19) can also be calculated first with the velocity u and then with the gradient of u. Therefore, equation (2) can be rewritten as equation (20).

$$\left((\rho\vec{v}) \cdot \vec{\nabla}\right)\vec{v} = \left((\rho u) \cdot \vec{\nabla}\right)u \quad (20)$$

As already mentioned in chapter (14), u in equations (8, 14, 15) and (18) stands for the velocity $\vec{v}$. Therefore, equation (20) can be rewritten as equation (21).

$$\left((\rho\vec{v}) \cdot \vec{\nabla}\right)\vec{v} = \left((\rho\vec{v}) \cdot \vec{\nabla}\right)\vec{v} \quad (21)$$

That means the second term from equation (17) and the second term from the equation 18 can be equated. However, it must be mentioned at this point that the second term from equation (17) has a minus signed. Whether and how this minus is relevant has to be discussed. Next, the third term from equation (17) $(\vec{v} \cdot \vec{\nabla})(\rho\vec{v})$ will be brought into a form that is similar to the form of the third term from equation (18) $(\rho\,\delta u/\delta t)$. To do this, the third term from equation (17) must first be written in column form. It should be noted that the gradient of a vector results in a matrix. Equation (22) shows how the third term from equation (17) must then be rewritten.

$$(\vec{v} \cdot \vec{\nabla})(\rho\vec{v}) = \begin{pmatrix} \frac{\delta\rho v_x}{\delta x} & \frac{\delta\rho v_x}{\delta y} & \frac{\delta\rho v_x}{\delta z} \\ \frac{\delta\rho v_y}{\delta x} & \frac{\delta\rho v_y}{\delta y} & \frac{\delta\rho v_y}{\delta z} \\ \frac{\delta\rho v_z}{\delta x} & \frac{\delta\rho v_z}{\delta y} & \frac{\delta\rho v_z}{\delta z} \end{pmatrix} \cdot \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} \quad (22)$$

Now, according to the rules of vector calculation, the resulting gradient $\left(\vec{\nabla}(\rho\vec{v})\right)$ is calculated with the velocity vector $\vec{v}$ as a general solution (for all substances).

$$(\vec{v}\cdot\vec{\nabla})(\rho\vec{v}) = \begin{pmatrix} \frac{\delta(\rho v_x)\cdot v_x}{\delta x} + \frac{\delta(\rho v_x)\cdot v_y}{\delta y} + \frac{\delta(\rho v_x)\cdot v_z}{\delta z} \\ \frac{\delta(\rho v_y)\cdot v_x}{\delta x} + \frac{\delta(\rho v_y)\cdot v_y}{\delta y} + \frac{\delta(\rho v_y)\cdot v_z}{\delta z} \\ \frac{\delta(\rho v_z)\cdot v_x}{\delta x} + \frac{\delta(\rho v_z)\cdot v_y}{\delta y} + \frac{\delta(\rho v_z)\cdot v_z}{\delta z} \end{pmatrix} = \vec{x}_{(v,\rho)} \tag{23}$$

The result is a new vector $\vec{x}_{(v,\rho)}$. This is shown in equation (23). For substances that are not subject to any deformation and have a homogeneous density, equation (24) applies.

$$(\vec{v}\cdot\vec{\nabla})(\rho\vec{v}) = \begin{pmatrix} \frac{\delta(\rho v_x)}{\delta x}\cdot v_x + 0 + 0 \\ 0 + \frac{\delta(\rho v_y)}{\delta y}\cdot v_y + 0 \\ 0 + 0 + \frac{\delta(\rho v_z)}{\delta z}\cdot v_z \end{pmatrix} = \vec{x}_{(v,\rho)} \tag{24}$$

The expression from equation (24) is simplified to equation (25).

$$(\vec{v}\cdot\vec{\nabla})(\rho\vec{v}) = \begin{pmatrix} \frac{\delta(\rho v_x)}{\delta x}\cdot v_x \\ \frac{\delta(\rho v_y)}{\delta y}\cdot v_y \\ \frac{\delta(\rho v_z)}{\delta z}\cdot v_z \end{pmatrix} \tag{25}$$

In the case of Newtonian liquids under constant pressure conditions, the mass occupancy is constant and is interpreted as density ρ. Therefore, it can be excluded as a factor on the right-hand side of equation (25). This results in equation (26).

$$(\vec{v}\cdot\vec{\nabla})(\rho\vec{v}) = \rho \begin{pmatrix} \frac{\delta v_x}{\delta x}\cdot v_x \\ \frac{\delta v_y}{\delta y}\cdot v_y \\ \frac{\delta v_z}{\delta z}\cdot v_z \end{pmatrix} \tag{26}$$

Now, on the right-hand side from equation (26), the velocity $\vec{v}$ is derived from the distance $(\delta\vec{v})/(\delta\vec{s})$ Equation (27) arises.

$$\frac{\delta\vec{v}}{\delta\vec{s}} = \frac{\delta}{\delta t} \tag{27}$$

The expression on the right-hand side from equation (27) is now substituted into equation (26). Assuming that the term $\vec{v}$ is equated with the term u equation (28) is the result.

$$(\vec{v}\cdot\vec{\nabla})(\rho\vec{v}) = \rho \begin{pmatrix} \frac{\delta}{\delta t}\cdot v_x \\ \frac{\delta}{\delta t}\cdot v_y \\ \frac{\delta}{\delta t}\cdot v_z \end{pmatrix} = \rho \begin{pmatrix} \frac{\delta v_x}{\delta t} \\ \frac{\delta v_y}{\delta t} \\ \frac{\delta v_z}{\delta t} \end{pmatrix} = \rho\left(\frac{\delta\vec{v}}{\delta t}\right) = \rho\frac{\delta u}{\delta t} \tag{28}$$

The result from equation (28) now corresponds to the third term from equation (18). That means, that the third term from equation (17) is equated to the third term from equation 18.

$$(\vec{v}\cdot\vec{\nabla})(\rho\vec{v}) = \rho\frac{\delta u}{\delta t} \tag{29}$$

The fourth term from equation (17) is written, as $\vec{v}\left(\vec{\nabla}\cdot(\rho\vec{v})\right)$ and the fourth term from equation (17) is written, $\vec{\nabla}\cdot(\sigma_{(u,p)})$. The term $\sigma_{(u,p)}$ stands for the mechanical normal stress [6], which here depends on the velocity u and the pressure p. It is defined as the viscous stress tensor (30).

$$\sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{pmatrix} \tag{30}$$

Applying the divergence to this tensor creates a vector, i.e. a tensor of the first degree. This is shown in equation (31).

$$\vec{\nabla}\cdot\sigma = \begin{pmatrix} \frac{\delta\sigma_{11}}{\delta x} + \frac{\delta\sigma_{12}}{\delta y} + \frac{\delta\sigma_{13}}{\delta z} \\ \frac{\delta\sigma_{21}}{\delta x} + \frac{\delta\sigma_{22}}{\delta y} + \frac{\delta\sigma_{23}}{\delta z} \\ \frac{\delta\sigma_{31}}{\delta x} + \frac{\delta\sigma_{32}}{\delta y} + \frac{\delta\sigma_{33}}{\delta z} \end{pmatrix} = \begin{pmatrix} \sigma_{\text{a div}} \\ \sigma_{\text{b div}} \\ \sigma_{\text{c div}} \end{pmatrix} \tag{31}$$

The vector resulting from the $\vec{\nabla}\cdot\sigma$ has the physical unit $\frac{g}{\overrightarrow{m}\cdot s^2} = \vec{F}$. With this unit, the dependence of σ can be mapped, under certain circumstances, on both the speed u and the pressure p. That is why σcan also be written for $\sigma_{(u,p)}$.

The fourth term from equation (17) shows the following relationship, $\vec{v}\left(\vec{\nabla}\cdot(\rho\vec{v})\right)$. In this context, the $\vec{\nabla}\cdot(\rho\vec{v})$ provides a purely numerical value and a physical unit. This can be seen from equation (32).

$$\vec{\nabla}\cdot(\rho\vec{v}) = \frac{\delta(\rho v_x)}{\delta x} + \frac{\delta(\rho v_y)}{\delta y} + \frac{\delta(\rho v_z)}{\delta z} \tag{32}$$

If the scalar expression from equation (32), however, multiplied by the velocity $\vec{v}$, as shown in the fourth term from equation (17) is required, however, a vector results. This relationship is shown in equation (33).

$$\vec{v}\left(\vec{\nabla}\cdot(\rho\vec{v})\right) = \begin{pmatrix} v_x \cdot \left(\frac{\delta(\rho v_x)}{\delta x} + \frac{\delta(\rho v_y)}{\delta y} + \frac{\delta(\rho v_z)}{\delta z}\right) \\ v_y \cdot \left(\frac{\delta(\rho v_x)}{\delta x} + \frac{\delta(\rho v_y)}{\delta y} + \frac{\delta(\rho v_z)}{\delta z}\right) \\ v_z \cdot \left(\frac{\delta(\rho v_x)}{\delta x} + \frac{\delta(\rho v_y)}{\delta y} + \frac{\delta(\rho v_z)}{\delta z}\right) \end{pmatrix} \tag{33}$$

The resulting vectors from equation (33), and from equation (31), both have the physical unit $\frac{g}{\overline{m}\cdot s^2} = \vec{F}$. In addition, the resulting vector from equation (33) is also dependent on the pressure p and the velocity u / $\vec{v}$. The next thing in common is that both vectors make a statement about the tensions within a substance. For these reasons we can use the term four from equation (17) and the term four from equation (18) equate. This result is shown in equation (34).

$$\vec{v}\left(\vec{\nabla}\cdot(\rho\vec{v})\right) = \vec{\nabla}\cdot(\sigma_{(u,p)}) \tag{34}$$

Term 5 from Equation (17) is $(\rho\vec{v})(\vec{\nabla}\cdot\vec{v})$ and Term 5 from Equation is 0. Equation (8) says that $\vec{\nabla}\cdot\vec{v} = 0$ is. Inserting equation 8 into equation (17) results in the expression from equation 35. This means that the fifth term from equation (17) can be equated with the fifth term from equation (18). This can also be seen from equation (35).

$$(\rho\vec{v})(\vec{\nabla}\cdot\vec{v}) = (\rho\vec{v})\cdot 0 = 0 \tag{35}$$

Since the first term of equation (18) (f) is not precisely defined, it can be calculated with the first term from equation (17) $\left(\vec{\nabla}\times(\vec{v}\times(\rho\vec{v}))\right)$ are equated. Equation (36) results by equating the first term from equation (17) and the first term from equation (18).

$$\vec{\nabla}\times(v\times(\rho\vec{v})) = f \tag{36}$$

Here, too, there is a minus sign in the first term of equation 17. For this term, too, it must be discussed whether and what effects this sign has on equation (36).

DISCUSSION

1. It remains to be discussed whether this expression $(\vec{\nabla}\cdot\vec{v}) = 0$ is valid for all substances, including those that are not subject to Newton's laws. The problem is that the relationship from equation 37 holds.

$$(\vec{\nabla}\cdot\vec{v}) = (\mathrm{Sp})\left(\vec{\nabla}\,\vec{v}\right) \tag{37}$$

If the relationship from equation (37) gold plated, then $(\mathrm{Sp})\left(\vec{\nabla}\,\vec{v}\right) = 0$ must also apply. The question here would be what effect this would have on the two equations (17) and (18).

2. What effects would an inhomogeneous density distribution of a substance have on the solution approach? Do other rules of vector calculus apply in this case?

3. Based on 2, what effects would it have if the mass occupancy was included in the solution as a vector quantity instead of the density?

4. Is the approach from equation (4) a fundamental law of nature that is valid for all substances?

5. With reference to the question to (4), which state of aggregation then have physical fields?

6. Term one $\left(\vec{\nabla}\times(\vec{v}\times(\rho\vec{v}))\right)$ and term two $\left(-\left((\rho\vec{v})\cdot\vec{\nabla}\right)\vec{v}\right)$ from equation (17) have a minus sign. It remains to be discussed whether and what effect this has on equating the two equations (17) and (18).

CONCLUSIONS

Through the mathematical connection to the vector calculation and the physical-mathematical connection to the flow law of electrodynamics, the general validity of the Navier-Stokes equations could be adequately described. The fact that the vector calculation can create a connection from the unipolar induction and thus the "Maxwell equations" to the Navier-Stokes equations allows the conclusion that this approach is a fundamental field equation for the dynamics of all substances from fields Depicts gases and liquids, as well as solids and other unknown substances. It also remains to be determined whether the Smoluchowsky equations [7] can also be described using equation (4). For comparison, all equations relevant for this elaboration are written below. This shows the connection between the "Maxwell equations" and the Navier-Stokes equations announced in Chapter one of this paper. Calculation rule from vector calculation:

$$\begin{aligned}\vec{\nabla}\times\vec{c} &= \vec{\nabla}\times\left(\vec{a}\times\vec{b}\right)\\ &= \left(\vec{b}\cdot\vec{\nabla}\right)\vec{a} - \left(\vec{a}\cdot\vec{\nabla}\right)\vec{b} + \vec{a}\left(\vec{\nabla}\cdot\vec{b}\right)\\ &\quad - \vec{b}(\vec{\nabla}\cdot\vec{a})\end{aligned} \tag{11}$$

possible fundamental field equation:

$$\begin{aligned}\vec{\nabla}\times\overrightarrow{\Phi_1} &= \vec{\nabla}\times\left(\vec{v}\times(a\overrightarrow{\Phi_2})\right)\\ &= \left((a\overrightarrow{\Phi_2})\cdot\vec{\nabla}\right)\vec{v} - \left(\vec{v}\cdot\vec{\nabla}\right)(a\overrightarrow{\Phi_2})\\ &\quad + \vec{v}\left(\vec{\nabla}\cdot(a\overrightarrow{\Phi_2})\right) - (a\overrightarrow{\Phi_2})(\vec{\nabla}\cdot\vec{v})\end{aligned} \tag{4}$$

Unipolar induction:

$$\vec{\nabla} \times \vec{E} = \vec{\nabla} \times (\vec{v} \times \vec{B}) = (\vec{B} \cdot \vec{\nabla})\,\vec{v} - (\vec{v} \cdot \vec{\nabla})\,\vec{B} + \vec{v}\,(\vec{\nabla} \cdot \vec{B}) - \vec{B}\,(\vec{\nabla} \cdot \vec{v}) \quad (3)$$

"Maxwell equations" according to Heaviside:

$$\vec{\nabla} \times \vec{E} = \vec{\nabla} \times (\vec{v} \times \vec{B}) = 0 - \frac{\delta \vec{B}}{\delta t} + 0 - 0 \quad (38)$$

with

$$(\vec{\nabla} \cdot \vec{v}) = 0 \quad (39)$$

Electric field equation according to Dirac

$$\vec{\nabla} \times \vec{E} = \vec{\nabla} \times (\vec{v} \times \vec{B}) = 0 - \frac{\delta \vec{B}}{\delta t} + \vec{v}\,(\vec{\nabla} \cdot \vec{B}) - 0 \quad (40)$$

with

$$(\vec{\nabla} \cdot \vec{B}) = \rho_m \quad (41)$$

Navier-Stokes equations:

$$f = \rho \frac{\delta u}{\delta t} + \rho(u \cdot \vec{\nabla})\,u - (\vec{\nabla} \cdot \sigma_{(u,p)}) + 0 \quad (7)$$

with

$$(\vec{\nabla} \cdot u) = 0 \quad (8)$$

The comparison of all equations indicates a common mathematical basis. From a physical point of view, it seems that the velocity vector plays a role in calculating the movements of fields and substances.

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