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Foundations of Quantum Computing: I-Demystifying Quantum Paradoxes

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ABSTRACT

Speedy developments in Quantum Technologies mandate that fundamentals of Quantum Computing are well explained and understood. Meanwhile, paradigms of so-called quantum non-locality, wave function (WF) "collapse", "Schrödinger cat" and some other historically popular misconceptions continue to feed mysteries around quantum phenomena. Arguing that above misinterpretations stem from classically minded and experimentally unverifiable perceptions, recasting Principle of Superposition (PS) and key experimental details into classical notions. Revisiting main components of general quantum measurement protocols (analyzers and detectors), and explaining paradoxes of WF collapse and Schrödinger cat. Reminding that quantum measurements routinely reveal correlations dictated by conservation laws in each individual realization of the quantum ensemble, manifesting "correlation-by-initial conditions" in contrast to traditional "correlation-by-interactions". We reiterate: Quantum Mechanics (QM) is not a dynamical theory in the same sense the Classical Mechanics (CM) is – it is a statistical phenomenology, as established in 1926 by Born's postulate. That is, while QM rests on conservation laws in each individual outcome, it does not indicate how exactly a specific outcome is selected. This selection remains fundamentally random and represents true randomness of QM, the latter being a statistical paradigm with a WF standing for a complex-valued distribution function. Finally, PS is the backbone of a quantum measurement process: PS can be conveniently viewed as a composition of partial distributions into the total distribution – similar to classical probability mixtures – and is effectuated experimentally by the analyzer part of a measuring device.

Keywords: Quantum Mechanics; Quantum Computing; Quantum Paradoxes; Classical Mechanics; Wave Function; Principle of Superposition; Copenhagen Interpretation; Wave-Particle Duality

INTRODUCTION

Quantum Mechanics (QM) will soon celebrate its centennial jubilee and stellar accomplishments in the 20st century, but the wave of paradoxes and interpretations and misinterpretations! - do not show signs of subsiding. What's more, latest developments in quantum computing and prior to that the probe of Bell's hidden parameters theorem-only stimulated the interest to fundamental quantum features of superposition and entanglement, which, in turn, renewed the curiosity regarding the nature and underpinnings of quantum paradoxes, such as quantum non-locality and spooky action at the distance in the Einstein-Podolsky-Rosen (EPR) paradox, wave function (WF) collapse, Schrödinger cat and some others. While these paradoxes were discussed and well explained to the bones in the past within a consistent framework of QM, lots of this work has been simply either forgotten or missed to date.

This is particular the case for intense discussions between Bohr and Einstein about the foundations of QM, the detailed analysis and explanation of EPR paradox via statistical interpretation of QM in late 1930s, 1950s and 1960s, the probe of Bell's theorem on hidden parameters in QM in 1970s and 1980s, and so on. In addition, detailed argumentation via main caliber mathematical guns of QM appeals mostly to specialists in quantum physics, but is totally inaccessible to professionals in other fields and general public.

Meanwhile, current intense studies in quantum computing will most likely result in the near future in steady expansion of the use of quantum computers, and their successful application will be substantially premised on understanding of QM foundations by prospective users. Ironically, none other than R. Feynman, one the most brilliant physicists of all times, whose impact on quantum theory is just impossible to overvalue-contributed to the glory of QM mysteriousness by once commenting: "…But I think I can safely say that nobody understands quantum mechanics" [1]. Despite an obvious eloquence, this figure of speech is a bit far-fetched: paraphrasing in this regard L.I. Mandeshtam (1939), while some questions in QM are yet to be answered, and lots of work still lies ahead, for some other issues clarifications can be made. Accordingly, in the brief notes below, we attempt to clarify the most vexing and mysterious quantum paradoxes without resorting to heavy mathematical language of QM and by appealing to the common sense and heuristic arguments.

A few essential mathematical technicalities are moved to the technical appendices at the end. Specifically, Section 1, gives very brief introduction to QM in comparison to Classical Mechanics (CM). In Section 2 we discuss Born's Statistical Postulate and the Principle of Superposition (PS). Section 3 describes general Quantum Measurement techniques in the context of PS and explains the paradoxes of WF collapse and Schrödinger cat.

Finally, Section 4 addresses long distance quantum correlations in many-body systems and demystifies the so-called quantum non- locality and spooky action at the distance. Finally, to broaden a prospective audience and in interests of brevity, the exposition here and there was made somewhat cursory. We hope though that non-specialists should be able to follow key points and main conclusions. Readers familiar with only very basics of quantum theory are encouraged to read most of the text.

We intend to revisit further underpinnings of quantum mechanics in more detail sometime later on. For compactness, the following intuitive abbreviations are used for most repetitive terms: **CM**-classical mechanics, **QM**-quantum mechanics, **PS**-principle of superposition, **SE**-Schrödinger equation, **PA**-probability amplitude, **WF**-wave function, **EPR**-Einstein-Podolsky-Rosen, **CI**-Copenhagen Interpretation, **WPD**-wave-particle duality.

BASICS OF QUANTUM MECHANICS VS CLASSICAL MECHANICS

We will not use popular slogans aka "Quantum physics for dummies" and the like, but merely summarize some minimal facts which should help even technically non-savvy readers gain some comfort with quantum basics. Also, from time to time, we make comments for readers with some experience in quantum physics just to avoid pure handwaving and maintain the solid foundations. These comments though can be skipped at will. With that, in CM the motion of a classical object is fully determined by its coordinate x and linear momentum p (velocity): that is, given initial pair of x and p at $t = 0$, (Cauchi problem), the object moves along the unique line called trajectory, which is a time-sequence of (x, p) pairs. This is, essentially, a classical determinism. Stated differently, there exists one and one only trajectory between any x_1 and x_2, given initial or final velocities. The form of the trajectory is governed by the Newton Second law (of more comprehensively, Lagrange or Hamilton equations) (see Appendix 1).

In contrast, in QM, x and p do not co-exist: a particle can have precise x or p, but not both at the same time. This is Heisenberg uncertainty principle: that is, the product of coordinate and momentum uncertainties $\Delta x \Delta p > \hbar/2$, hence, trajectories in QM do not exist. $\hbar$ here stands for the celebrated Planck constant, which effectively separates the classical and quantum worlds. Therefore, between any two points x_1 and x_2, there exists not a unique trajectory, but only a probability (more precisely, probability amplitude, (PA)) of getting from x_1 to x_2 in time t. This is known as Feynman PA $K(x_1, x_2, t)$ in the Feynman formulation of QM. (Feynman formulation posits, using the classical language, that between two given points x_1 and x_2 , there exist infinitely many trajectories - known as Feynman paths - which all contribute to the total K).

The probability itself is then $|K(x_1, x_2, t)|^2$. If we do not care about where the particle moved to x_2 from, we can drop x_1, and then K becomes $\psi(x_2, t)$, known as Schrödinger wave function ψ, meaning the PA of a particle having x around x_2. Both K and ψ are governed by the same Schrödinger equation (SE) in more traditional Schrödinger formulation of QM. Similar to $|K|^2$, the probability of finding the particle around x_2 is $|\psi(x_2, t)|^2$. And, as any normal probability, it meets normalization condition that the total probability of finding a particle anywhere is equal 1. This is the so called Born's statistical postulate (BSP), which is a top jewel of the whole QM, and which interpretation we will return to later. (Further below we omit index 2 next to x in WFs). The SE and BSP imply that, while the causality in CM is individual, (that is, CM describes classical object motion over individual trajectory starting from initial position), QM does not handle single events, but rather probability of events via a quantum ensemble of events. In other words, the causality in QM is statistical, not individual. And this statistically is not inflicted by our lacking of some information as in Classical Statistical Mechanics, but because it is the very nature of the micro-world and QM. Technically, it is manifested by the fact that given WF ψ at the initial moment $t = 0$, the SE provides all further evolution for it for any $t > 0$. This is the key pillar of the QM view of the micro-world and we'll revisit it several times statisticality later. Readers familiar with QM basics are then invited to review the Appendixes 1 and 2.

BORN'S STATISTICAL POSTULATE AND PRINCIPLE OF SUPERPOSITION

According to the initial de Broglie conjecture, WFs were deemed as some material waves associated with real particles. This had very much influenced the so-called Copenhagen Interpretation (CI). When Born devised his Statistical Postulate, the WF became a strange hybrid of a material wave with probabilistic properties, which caused lots of troubles to CI. Among other things, it leads to a number of paradoxes, the most famous of which is the "collapse" of WF. However, over the years, it became clear, that WF is not a material wave, but a wave of probability, so to speak. This was among factors, that prompted Feynman to introduce his interpretation of ψ as probability amplitude, complex-valued function, with square modulus $|\Psi(x,t)|^2$ being a normal classical probability.
Now, if we decompose then a WF, say, in x-space, over eigen functions in momentum p-space, we then see, that PA in x-space is the weighted sum of PAs in p-space, and vice versa. What's more, this duality applies to ANY pair of spaces, depending on the convenience of the problem tractability.
This leads us to the fundamental Principle of Superposition (PS) in QM. In its simplest form it states that for any two solutions of Schrödinger equation ψ_1 and ψ_2, their weighted sum $(c_1\,\psi_1 + c_2\,\psi_2)$ is also a solution. Obviously, this is a trivial consequence of SE linearity. Recognizing now that the number of components is not limited, we now see that PS is merely a decomposition of PAs/WFs in one space over PAs/WFs in another space. And it echoes the similar situation in classical probability. Indeed, the classical probability distribution (PD) in one space can be easily converted to PD in another space given the relationship between independent variables and normalization condition. The key difference with QM here, is that in QM we operate with complex-valued PA, not probability itself. Considering Born's postulate, we, therefore, can alternatively interpret PAs as pseudo-probability distribution (for brevity, below we typically omit pseudo), which brings a lot of heuristic similarity with classic probabilities, and clarity when applying PS.
We will discuss this in more detail at the end of next section, Section 3. This now enables the concept of ensemble, or quantum ensembles (QE), coined first by Von Neumann[2], and the so-called statistical interpretation of quantum mechanics, advanced mostly by L.E. Ballentine [3], D.I. Blokhintsev [4], Leonid Isaakovich Mandelstam[5], K.V. Nikolsky [6] in late 30s – early 40s of 20st century. The essence of QEs and statistical interpretation follows in the next section.

PRINCIPLE OF SUPERPOSITION AND QUANTUM MEASUREMENTS

In this section we introduce some heuristics helpful in connection with PS and quantum measurements. First of all, according to SE, micro-particles lead their daily in and out life at the level of WFs $\psi(x)$, that is, quantum amplitudes. Quite provisionally, we can call this level virtual, or underground level, invisible readily to the observer with macroscopic measuring devices. (We note in passing, that the discussion what's real vs virtual in QM is as old as QM itself.
In particular, the famous epigraph to this paper above by Albert Einstein about the Moon came from the well-known discussion between A. Einstein and A. Pais. J. Wheeler [7, 8] even went as far as saying "No elementary quantum phenomenon is a phenomenon until it is observed phenomenon"). Only occasionally, when we perform actual measurements, they-microparticles-step up above the ground to the macro-level manifested by $|\Psi(x)|^2$. We emphasize here, that micro-particles lead their virtual life regardless and independent of whether we observe them by performing the measurements. It is in that sense that Einstein once asked A. Pais "whether he really believed that the Moon existed only when he looked at it "(!). Our answer to that is, in a sense, that the world of micro-particles is as objective as it gets at its virtual level, regardless of our measuring intervention. The virtual world is merely disconnected from the real macro-world until and unless we perform measurements, but that by no means negates its reality.
We mention here in passing that once we measure $|\psi(x)|^2$, we can also restore $\psi(x)$ itself (up to an arbitrary global phase factor $exp(ia)$), but that's calls for some non-trivial translation from $|\psi(x)|^2$ level to $\psi(x)$ level. With that preamble we now turn to basics of quantum measurement practices.
We will be considering only direct measurements, when the system is readily subjected to the measuring action (as opposed to indirect measurements, when the system gets in touch with another system, and that second system is a subject of an actual measurement). Consider a beam of particles, say, electrons or atoms, described by some general WF $\psi(x)$.
We would like now to measure p values (linear momentum) for particles in our $\psi(x)$ beam. Considering that this $\psi(x)$ can be expanded over eigen WFs with specific momentum p as $\psi(x) = \Sigma f_p\,\psi_p(x)$ we say that the momentum of the whole $\psi(x)$ is indetermined, and $\psi(x)$ is in a superposition state over different momentum p state. Oftentimes though, eigen-functions $\psi_p(x)$ closely overlap and are inconvenient for immediate measurements.
We can therefore subject the beam to diffraction grating – the latter can be thought of as some kind of external field –

which separates $\psi_p(x)$ in space. These changes $\psi(x)$ to $\varphi(x) = \sum g_p \varphi_p(x)$, but the relationship between $\psi_p(x)$ and $\varphi_p(x)$ is known, as well as between f_p and g_p . (quite often $f_p \approx g_p$). These new $\varphi_p(x)$ are now well separated and make up independent "channels" for particles. If we now place detectors at the end of each channel, clicking of the detector would register a particle propagated through the particular p-channel. Counting clicks in each channel gives us $|g_p|^2$, and therefore, $|f_p|^2$.

This whole arrangement is depicted in Fig. 1 (borrowed from: D.I. Blokhintsev [9], Statistical ensembles in quantum mechanics/In: Quantum mechanics, determinism, causality, and particles, Reidel [10]). Typical examples of the arrangement on Fig.1 include, but by no means are limited to, splitting the initial particle beam by subjecting it to scattering off a crystal or to non-uniform magnetic field (as in the archetypal Stern-Gerlach experiment demonstrating the effects of electron spin).

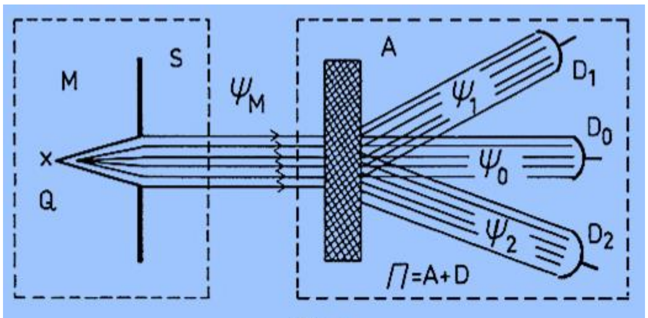

Fig. 1.

On Fig.1. only three channels are displayed. ψ_M is an initial beam $\psi(x)$, A stands for an analyzer, D_0, D_1, D_2 – detectors, corresponding to eigen functions $\varphi_1(x)$, $\varphi_2(x)$, $\varphi_3(x)$ (Fig.1 assumes $\varphi_i(x) = \psi_i(x)$). Here the stage of propagating through the channels before detector clicking is called an analyzing stage. It is followed then by the detector stage – detector's clicking.

Importantly, the stage of an analyzer A performs just a spectral decomposition of initial WF over eigen functions $\varphi_p(x)$ and as such is fully reversible, because the coherency of $\varphi_p(x)$ is preserved within $\phi(x)$. That is, we can apply another external field, essentially, compensating for the impact of the initial field, to restore initial WF $\psi(x)$. In other words, before being registered by detectors, all particles belong to the coherent ensemble of WF $\phi(x)$, i.e all particles are still in a superposition state over p. On the contrary, detector stage is irreversible: the momentum of each particle is known with certainty and particles no longer constitute the common ensemble $\phi(x)$. This second stage is referred to in the literature as a so-called collapse of the superposition state WF $\phi(x)$ to the state with certain single p.

If WF is interpreted as dynamic variable connected to the individual particle, then the collapse jump presents an obvious logical difficulty and paradox to CI. In contrast, in Statistical Interpretation the WF is merely a distribution function (pseudo-distribution, that is)-just a statistical metric for electron beam ensemble-and there is no collapse of any dynamic variable. The change of the distribution function AFTER the detector action is now just an obviously simple mathematical fact, very well known in classical probability theory. Further, in relation to PS, the plus sign in the PS formula is only a foot-print of the linearity of SE.

However, it is not infrequently that particle being in the superposition state is interpreted as residing simultaneously in all superposition components. This interpretation runs into substantial difficulties, because now one needs to explain how this simultaneity should be understood, and, correspondingly, experimentally probed.

In contrast, the interpretation of WF as a distribution function is free of that weakness. Indeed, the moment we interpret PA as a (pseudo-) distribution function, the summation in PS immediately becomes a law of composing partial distribution functions $\psi_p(x)$ into a total one, $\psi(x)$. This interpretation has its immediate analogy in a composition rule for classic probability mixtures by Bean [11].

Further, in connection with Section 2 and in the context of Fig.1 arrangement, before the detector clicking, we do not know what the momentum p of the particle is. We just say, that the odds of the particle having specific p are, somewhat loosely speaking, proportional to appropriate PA f_p, very much similar to a classic probability mixture (in this context we disregard for now that PA is complex-valued). Each particle in the beam/ensemble brings its own contribution into forming a total distribution function via a build-up of spots on a photo-sensitive screen (or screen, covered by dense array of detectors D_i), that is, the total probability distribution $|\psi(x)|^2$ emerges via a consecutive arrivals of beam particles at the screen. Importantly here, that PA proves complex-valued (it can be written as $(P)^{1/2}exp(i\alpha)$, where P- classical probability, α – phase). That is, the true randomness in QM is more involved than just a one-dimensional classical probability: at the virtual/amplitude level the traditional amplitude modulus, $(P)^{1/2}$, is complemented by an additional parameter, phase α, to form a complex-valued pseudo-distribution function, $PA = (P)^{1/2}exp(i\alpha)$.

And to transit to macro-level, we take $|PA|^2$. It is this phase α, the hallmark of the coherence, which enables the phenomenon of interference. And accordingly, when passing a detector, particles lose their ability to interfere because the phase is destroyed by an irreversible act of detection. For further details on detector operations see [4, 9].

It is worth mentioning here, that historically (and conceptually!) the idea of channelling particles via different paths in quantum measurements was essentially implemented first in the celebrated Young two-slit experiment, Fig.2 [12].

In it the beam of particles (photons, electrons, atoms, etc.), after passing the first screen with two narrow slits forms – under proper conditions! - an interference intensity pattern on a second screen, rather than just a simple sum of intensities emanating from slits 1 and 2, hinting at a wave nature of micro-particles. When the beam intensity is very low, the interference appearing on the second screen sheds a lot of light on the nature of the wave-particle duality and this is exactly what gave R. Feynman reason to brand this experiment as a heart of quantum mechanics. In conclusion of this section, we briefly touch the Schrödinger cat paradox. Say, we have only two channels on Fig.1, and at the end of one channel the detector harm a poor cat, and at the other – does not. Will then the cat be in the superposition state? Obviously, not!

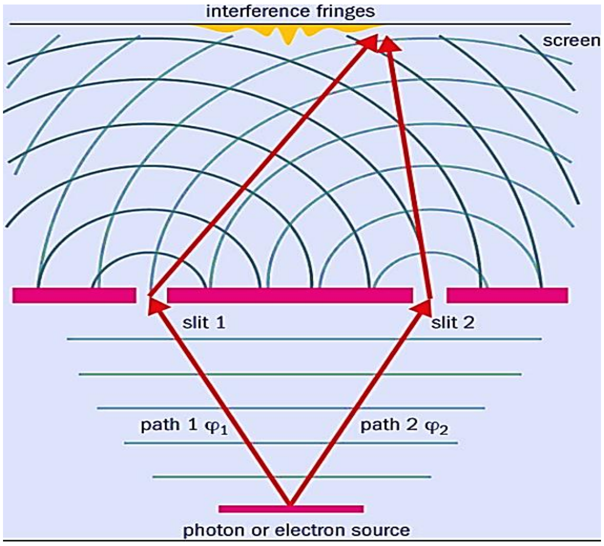

Fig.2

As was explained above, the superposition coherency is preserved only ***BEFORE*** detectors, not after. The detector clicking destroys the coherency and converts the initial WF into the incoherent mixture. This mixture is a well-known mathematical object in classical probability and does not bring a cat in any superposition state, as expected [4]. In closing this section, we point out that our modern understanding of quantum measurements owes a lot to tireless efforts of D.I. Blokhintsev [3], whose scientific and pedagogical talent made his papers and books (see the references by the end of the paper) on foundations of QM and measurement processes an invaluable legacy for generations of physicists to come.

CONSERVATION OF QUANTUM CORRELATIONS IN MANY-BODY SYSTEMS AND SPOOKY ACTION AT THE DISTANCE

Conservation laws, in particular, energy conservation principle, went through some difficult/testing times in relation to emerging quantum physics in 1920s. In connection to certain difficulties of quantum radiation theory in 1924 none others than Bohr (!), the founding father of quantum physics, along with Kramers and Slater (another two prominent theorists in quantum theory) hypothesized that conservation of energy in quantum mechanics happens only statistically, but not in each individual event (i.e. energy is preserved only on average, in the limit of $N \to \infty$, where N is the number of micro-particles undergoing quantum processes). The similar tendency was also noticed in Dirac's and Landau's comments regarding beta-decay of nuclei and internal structure of stars (both Dirac and Landau – undisputed geniuses of theoretical physics of 20st century!). In fairness, it should be noted that in light of Born's statistical postulate the very idea of statistical nature of energy conservation in QM - even if from somewhat superficial viewpoint-did not appear all too logically insane at the time. Fortunately, by joint efforts of experimentalists and theorists, the energy conservation prevailed by the end 1930s, and re-established itself as an unconditional backbone of the world the way we know it.

Another confirmation of conservation laws came in remarkable experiments of Freedman and Clauser in 1970s and Aspect's group in 1980s with correlated pairs of photons (born via radiative decay of doubly excited calcium atoms), aiming at probing Bell's theory of hidden parameters in QM. In recent years similar experiments were repeatedly conducted for various distances between correlated photons, and conservation laws were invariably upheld for each and every photon pair. These results stimulated a popularity of the so-called non-local interpretation of quantum mechanics: that is, there allegedly exists an omnipresent and instantaneous quantum interaction between elementary particles at any distance, which was eloquently branded by Einstein as a spooky action at the distance. This persistent correlation at any distance was termed as entanglement in quantum mechanics and, actually, became an important tool in current experiments in quantum technology and quantum computers.

Without delving into endless discussions of this phenomenon in a professional and popular literature alike, we'll point out to the following. Entanglement is, in fact, by no means an exotic phenomenon in particular experiments. On the contrary, it routinely emerges in, say, all collision processes in atomic and nuclear physics, and in elementary particle physics. Every time the pair (or group of particles) emerges from the interaction process and propagates freely all along, its dynamic variables do not change and remain correlated according to conservation laws. That is, the total linear momentum, or angular momentum, or spin or polarization, etc. remains the same no matter how far the particles move from each other as long as they do not interact. This obvious fact follows immediately from the SE equation with zero interaction and was famously discussed in relation to completeness of quantum theory in the paper of Einstein, Podolsky and Rozen in 1936 (the so called EPR paradox of quantum mechanics)[13].

While this effect finds its consistent explanation in statistical interpretation of quantum mechanics (more specifically-via the concept of statistical ensembles of quantum mechanics), there is a pressing need to give a layman/pedestrian

explanation, why on Earth, the particles remain correlated at any distance, i.e. what is the underlying physical reason for that, given the statistical/random nature of quantum experiments.
The answer though lies quite on the surface. Indeed, the SE is the wave type of equation, that is why the initial name to QM was actually wave mechanics. And its wave image of free point-wise particle with, say, certain linear momentum p, is a plane wave with the same momentum, i.e $exp(\pm ipx/\hbar)$. But in contrast to particles, which are point-wise objects, plane waves are not limited in space, they are, so to speak, everywhere (x extends to plus and minus infinity) and there is no distance between them, regardless whether we experiment with photons in a laboratory or with photons coming from distant stars. In other words, there is no distance between plane waves (and not only plane waves!) in SE equation of QM.
Therefore, the plane waves are, loosely speaking, always next to each other and there is no problem for them to communicate to maintain a perfect correlation to meet conservation laws restrictions. More generally, this is a manifestation of the so-called wave-particle duality principle and complementarity in QM, introduced by Bohr in his discussions with Einstein. Wave-particle duality simply means that certain aspects of phenomena in quantum world can be explained via either classical, i.e. particle-like view, or wave-like view, ***but not both***. And further similar examples in QM are innumerable. For our case of distant correlated photons that means that the correlation between photons emerges at their birth via the SE equation, i.e. via wave-like process, and that correlation remains the same precisely because of the wave nature. That's why attempts to explain it via classical point-wise view do not work and are bound to fail.
These photon correlations are quantum/wave correlations at their heart, and it does not make sense to ask for particle-like explanation. In that context, invoking the so-called non-locality in quantum mechanics is nothing but naïve attempt of enforcing classical logic to pure quantum phenomenon and unsurprisingly implies some spooky and strange causes. For mathematical details of how these heuristics translate into wave mechanics workings in quantum measurements, see [4].

CONCLUSIONS

Let's us summarize some key takeaways here.
1- QM doesn't operate with individual events aka CM does, but rather with their ensembles / statistical collectives. In doing so, QM does not manipulate with dynamic variables themselves, but rather with probability amplitudes (pseudo-probabilities) of these variables.
2- Despite a statistical character of QM, correlations implied by conservation laws are enforced in any individual event.
3- Principle of Superposition is the backbone of a quantum measurement process and its statistical interpretation. Principle of Superposition can be conveniently viewed as just a composition law of partial distribution into the total distribution-very much similar to classical probability mixtures by Bean [11].
4- Long-distance correlations between non-interacting particles originate from initial conditions mandated by conservation laws and keep on all along via trivial workings of wave component of wave-particle duality. Invoking in this regard a mysterious quantum non-locality imitates a true quantum wave mechanism by a fictitious, and, therefore, redundant, classically-minded logic.
5- Wave function collapse is an obsolete term / figure of speech for changing the statistics of micro-particles by a detection process and does not represent a real collapse of any material substance. In a similar fashion, the detection intervention resolves a paradox of Schrödinger cat.
Here we note a historical tribute to the statistical nature of QM and measurements in QM received outstanding treatments and elucidation in groundbreaking works of L. Ballentine, D. Blokhintsev, D. Bohm, R. Feynman, L. Mandelshtam, J. von Neumann, and K. Nikolsky, to name just a few. Accordingly, we wholeheartedly recommend their works (please, see below) and references for more detailed studies.

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APPENDIX 1

Newton and Schrödinger Equations

Newton Second law and initial conditions are,

$$F = ma \text{ or } m\frac{d^2x(t)}{dt^2} = \frac{\partial U}{\partial x},\ x(0) = x_0,\ \text{and } P(0) = P_0$$

Schrödinger equation (SE) for both ψ and K functions and initial conditions

$$i\hbar\frac{\partial\Psi(x,t)}{\partial t} = H\Psi(x,t) \quad \text{or} \quad i\hbar\frac{\partial K(x_1,x_2,t)}{\partial t} = H\,K(x_1,x_2,t)$$

where H here is the total energy of the system, and $\psi(x,0) = \psi_0(x)$, and Normalization condition is

$$\int |\Psi(x,t)|^2 dx = 1$$

APPENDIX 2

Hilbert Space of Quantum Mechanics

WFs can be taken as a function of not necessarily coordinate x, but any other coordinate q, such as linear momentum p, angular momentum M, etc. All these $\psi(q)$ stand for WF in different representations, e.g. momentum representation, angular representation and so on. Further, following Von Neumann [2] suggestion, $\psi(q)$ can be considered as projections of vector ψ onto Hilbert orts q. In Dirac notations, vector ψ is written as $<\psi|$, and projection components $<\psi|q>$ give $\psi(q)$. (In the extant literature, and quite inconsistently, $\psi(q)$ is sometimes written as $<\psi(q)|$, which should not lead to confusion though). If q is not continuous, but discrete variable, such as spin, etc., then $\psi(q)$ becomes ψ_i, and the normalization condition then reads $\Sigma|\psi_i|^2 = 1$. In QM dynamic variables are not just regular algebraic variables, but operators. (E.g. in coordinate representation, x is still x, but p is d/dx, M is $d/d\varphi$, energy $E = p^2/2m$ is $(2m)^{-1}d^2/dx^2$ and so on). Each of these operators C has its own so called eigen functions ψ_i such that $C\psi_i = c_i\psi_i$. These functions form complete set, so that any function $\psi(q)$ can be decomposed as a weighted sum of these functions. So, each operator provides its own space for projecting space vector ψ and weights appearing in the decomposition of the WF in one space make up for the WF in the other. For example, the coordinate WF $\psi(x)$ can be decomposed over eigen functions of the momentum $\psi_p(x)$, that is $\psi(x) = \int c(p)\,\psi_p(x)dx$, and, therefore, $c(p)$ becomes a WF $\psi(p)$ in momentum space. And vice versa, since $\psi(p) = \int c(x)\,\psi_x{}^*(p)\,dp$, where * stands for a complex conjugation, then $c(x)$ becomes WF $\psi(x)$ in coordinate space and so on. According to Born's probabilistic postulate, all these $\psi(x)$, $\psi(p)$, etc. are probability amplitudes – PAs - in their respective spaces. All that prerequisite should facilitate an understanding of Born's statistical postulate and Principle of the Superposition (PS).