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Foundations of Quantum Computing: II. The Many Faces of Quantum Uncertainty

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ABSTRACT

In this Part II we focus on a few key elements of quantum mechanics essential for understanding of quantum technologies and computing. We begin with a subtle but important similarity between classical and quantum mechanics which is typically overlooked in favor of an apparent differences. Further, it is reminded that classical motion can be obtained via averaging over quantum distributions / wave functions and, conversely, quantum distributions can be recast as a superposition of virtual classical paths. Relatedly, we emphasize the importance of the case intermediate between classical and quantum mechanics – that is, quasi-classical mechanics. The above background facilitates additional insights and heuristics into the mechanisms of widely acclaimed long distance correlations in quantum mechanics and origins of the coherency in quantum ensembles in the context of wave-particle duality.

Keywords: Quantum Mechanics; Quantum Computing; Quantum Uncertainty

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INTRODUCTION

In this Part II we continue (Part I: Foundations of Quantum computing I. Demystifying quantum paradoxes) an honorary attempt to dissolve the haze of mystery around certain facets of Quantum Mechanics randomness and speak about it in a *normal layman* language. There is a caveat though: any classical / heuristic model for a truly quantum event is by necessity bound to some sort of surrogating and should be taken as such. Therefore, a prudent grain of caution is always recommended to avoid improper oversimplifications or even vulgarizations.

Accordingly, in Sec.1 *Quantum vs Classical Mechanics probabilities* we explore some subtle and often underappreciated similarity between Classical Mechanics (CM) and Quantum Mechanics (QM) in contrast to well acclaimed differences. The next Sec.2 *Ehrenfest equations and Feynman paths* discusses to what extent CM can be expressed in QM terms and vice versa, i.e., Ehrenfest equations and Feynman path integrals. In Sec.3 *Some general heuristics on long–distance correlations* we consider classically minded prototypes for long-distance correlations in QM. In Sec.4 *Interpretation of WFs and their superposition* we discuss the notion of wave functions and their interpretations. And, finally, in relation to wave-particle duality, Sec.5 *Genesis and evolution of the wave-particle duality* ponders possible mechanisms behind formation of quantum ensembles, in particular, in the context of emerging coherent patterns in experiments with low-intensity beams.

Two final comments are in order: most of formal technical arguments and details in support for the heuristics in the paper are omitted to broaden its accessibility for technically non-savvy readers. For the same reason the list of references is kept very much limited: an interested reader should consult any more or less comprehensive text on quantum mechanics. As in Part I, for compactness, the following intuitive abbreviations are used for most repetitive terms: **CM**–classical mechanics, **QM**–quantum mechanics, **PS**–principle of superposition, **SE**–Schrödinger equation, **PA**–probability amplitude, **WF–**wave function, **EPR**-Einstein-Podolsky-Rosen, **CI**–Copenhagen Interpretation, **WPD**–wave-particle duality.

QUANTUM VS CLASSICAL MECHANICS PROBABILITIES

As is well known (and pointed out in Part I of this paper), while in CM the motion takes place via paths (called trajectories) fully specified by an initial position and momentum x and px, in QM trajectories do not exist simply because the position and momentum cannot be specified simultaneously (Heisenberg uncertainty principle). That is, given initial coordinate x in QM, the future particle locations are not known with certainty, but only probabilistically. And here comes a subtle similarity between CM and QM which is widely underappreciated. Indeed, if we specify only initial x, leaving initial momentum px arbitrary, then even in CM the future paths are undefined. What's more, even if we specify both initial and final positions x and y, keeping initial px or final momenta py arbitrary, then there would still have existed a whole bunch of trajectories connecting x and y. A trivial example from elementary physics: projectile motion in a uniform gravitational field, i.e., the motion of a shell fired at some angle to horizon. In other words, in that respect CM and QM are quite similar. However, once we begin squeezing the range of possible initial px or final py momenta, i.e., when uncertainties Δpx or Δpy reduce (and this is where the similarity begins to break), so does the spectrum of classically available paths, either emanating from x or connecting x to y, so that in the limit of Δpx or $\Delta py \to 0$ we obtain a uniquely defined classical trajectory. Such a refinement is not at all possible in QM even conceptually, because of the uncertainty principle, and thus prohibiting trajectories in QM. We pointed out to this parallelism because it proves helpful for constructing classically inspired heuristics to seemingly mysterious / puzzling quantum phenomena.

Historical aside: to our knowledge, one of the first indications to that subtlety was made as early as in 1933/1934 by Yu.B. Rumer [1]. Once at that, we note in passing that the exposition of QM vs CM in this book is refreshingly clear and concise, yet comprehensive (as opposed to many formidable texts written later), echoing the same of the all-time quantum masterpiece by P.A.M.Dirac [2]. We wholeheartedly recommend both jewels to all interested readers.

EHRENFEST EQUATIONS AND FEYNMAN PATHS

Given the CM-QM similarity discussed in the previous Section, question arises to what extent it is possible, if at all, to view CM motion as averaged over QM distributions, and vice versa, QM in terms of the CM trajectories.

First of all, an averaging of the Schrödinger equation (SE) over a spatial coordinate led to the equivalent Ehrenfest equation, which reads as the modified Newton second law $ma = F$ + Quantum corrections (ψ), where Quantum corrections is a cumulative notation for additional terms, arising from quantum effects, and ψ is the wave function of a system. In other words, this equation can be alternatively viewed as averaging over quantum states spaced around some "mean" trajectory. Further, under normal conditions Quantum Corrections term is comparable with F in the Right-Hand Side (RHS) and, as expected, the standard Newton equation does not apply. However, when $\hbar$ (Reduced Planck constant) diminishes, the Quantum corrections term reduces commensurately and totally vanish in the limit of $\hbar \to 0$, recovering thereby the pure classical Newton equation $ma = F$. In other words, in the quasi-classical limit $\hbar \to 0$, a classical motion is contributed by few quantum states tightly packed around the system center of mass.

The construction of the opposite view, QM in terms of CM trajectories, follows from the R. Feynman milestone result: namely, Feynman showed that the quantum motion can be rendered, in a sense, as an interference of classical trajectories. Specifically, the probability amplitude of getting, say, from x to y, which normally stems from SE, can be alternatively, *but equivalently*, obtained by summing amplitudes along all classical paths from x to y. More precisely, if for each and every imaginable trajectory connecting x and y (and trajectories need not necessarily be real *physical* trajectories) we calculate an ordinary classical action S_k, then the sum $\Sigma exp[(i/\hbar)S_k(x,y)]$ over all trajectories (k is the summation index) gives a quantum amplitude $k(x,y,t)$, which otherwise would come as a solution to SE. Without delving into this any further, we point out only three key points. First, Feynman path sum (or integral) became a standard technical tool in the modern Quantum Field Theory (QFT). Second, similar to Ehrenfest equations, as $\hbar \to 0$, i.e. in a quasi-classical situation, all exponents in the sum wildly oscillate and effectively cancel each other, except for those corresponding to paths in the neighborhood of classic paths (where $S_k \approx 0$-stationary points of S_k). That is, in the classical limit, quantum amplitudes are dominated by classical paths and their vicinity, as expected. In other words, QM is possible to construct from CM trajectories, and, the other way around, CM motion naturally arises in the $\hbar \to 0$ limit of QM**.** Last, but not the least, a quantum motion can be perceived, at least heuristically, as happening over the web of *virtual* classical trajectories. Clearly, a transparency and heuristic appeal of Feynman path integral are rather irresistible.

To bottom line, the transition CM ←→ QM looks as follows. As we move from CM to QM, a classical trajectory splits into a tight bundle of paths which continue to diverge as $\hbar$ grows. Conversely, when $\hbar$ reduces, quantum/Feynman paths coalesce around classical trajectory, and eventually fully collapse on it in the limit $\hbar = 0$. In the intermediate region (traditionally known as quasi-classical) where the system is already not classical, but not yet fully quantum, quantum amplitudes (and probabilities) follow directly from classical actions obtained along classical trajectories (see, for instance, R. Feynman, A. Hibbs[3]).

SOME GENERAL HEURISTICS ON LONG – DISTANCE CORRELATIONS

In Part I, we touched base on long-distance correlations (resulting from conservation laws) as a true wave phenomenon via a well reputable concept of wave-particle duality. We then emphasized, that even though SE is a statistical equation, the conservation laws hold in quantum mechanics not statistically, but (surprisingly in some sense) in every individual outcome: we can dub this as a *detailed* conservation, rather than a statistical one. Here, we'll offer additional qualitative arguments that this detailed conservation is not surprising, but is, in fact, what to be naturally expected from the two-way CM←→QM heuristics.

Intending for a sort of classically minded prototype for quantum long-distance correlations, consider first a shell at rest exploding into two equal pieces. At any time (and distance!) after the explosion the total momentum remains 0, i.e. the momenta (angular momenta, spins, etc.) of each piece are equal and opposite, as long as there are no external actions. We can even imagine a sequence of random explosions, producing every time a directionally random distribution of fragments, but as long as they are pair-wise balanced, the conservation still holds for all random realizations in every possible direction. Therefore, if we view quantum amplitudes as a virtual superposition of classical events (in a sense of classical imitation of quantum ensemble), loosely speaking, a la Feynman paths superposition and not necessarily in a coordinate space, but in some suitable representation, then we can expect a detailed translation of classical conservation to the quantum world. Obviously, this classical heuristic is only a surrogate imitation of a true quantum reality, but it helps understand that consistency and a smooth transition between classical and quantum cases obviates the need for an artificial quantum non-locality.

Conversely, consider now a classical motion (in light of the Ehrenfest equation) via linearly weighting some tight quantum states. Since the conservation clearly holds for a classical motion, the quantum Ehrenfest averages should do the same. In turn, these averages are made up linearly from quantum states, independent of each other: hence, the conservation should hold individually for each quantum *event* contributing to classical averages.

In the region intermediate between CM and QM (called quasi-classical) both mechanics overlap and coexist so that quantum amplitudes (wave functions) are directly related to classical trajectories. The importance of the quasi-classical mechanics extends way beyond fertile heuristic analogies and technical relationships between CM and QM: it serves for their mutual cross-validation. By way of example: in the above classical model of randomly distributed fragments the long-distance correlation of debris in every possible direction follows immediately, while from the quantum view the randomness in quantum measurements historically contributed to a confusion and even to the so-called *quantum non-local* interpretation. However, once we recall the quasi-classical relationships between classical trajectories and quantum amplitudes, the connection of wave functions in any representation to classically balanced outcomes becomes transparent, and so does the conservation in any random realization in QM.

INTERPRETATION OF WFS AND THEIR SUPERPOSITION

According to the initial de Broglie conjecture, WFs were deemed as some *material* waves associated with real particles. This had very much influenced the so-called *Copenhagen Interpretation* (CI). When M.Born devised his Statistical Postulate, the WF became a strange hybrid of a material wave with probabilistic properties, which caused lots of troubles to CI. Among other things, it led to number of paradoxes, the most famous of which is the *collapse* of WF. However, over the years, it became clear, that WF is not a material wave, but a wave of probability, so to speak. This was among factors, that prompted Feynman to introduce his interpretation of Ψ as *probability amplitude*, complex-valued function, with square modulus $|\Psi(x,t)|^2$ being a *normal* classical probability. In more precise terms, the gradual build-up of the interference picture in low-intensity beam experiments in conjunction with Born's statistical postulate lends a direct support to a view at wave function as a *distribution function amplitude*.

From that angle, the superposition principle represents merely a composition law of distribution functions, corresponding to individual eigen states. Therefore, in a superposition state, a dynamic variable assumes various values, each occurring with frequency generally proportional to the amplitude of corresponding eigen state in the superposition (more precisely, proportional to modulus square of wave function amplitude). In other words, under this view the measuring device does not need to make a decision of choosing among available superposing components, but rather each sample of the system / each ensemble member arrives at the measuring device in quite a specific virtual state. Rephrasing this slightly, a superposition state is a set of eigen states, each occurring commensurately to its amplitude in the superposition. We emphasize the word *virtual* above: while we do not know how exactly quantum randomness *assigns* specific values of

a dynamic variable to each sample of the system, technically, as long as we operate with this *assignment* at the amplitude level (which is nothing but ordinary QM calculations), the results will be correct and consistent. To reiterate: *because we are unable to observe virtual events at the amplitude level, the term Assignment should not be taken literally, as if it were implying some traditionally measurable process. Rather, it stands as a vivid picturing of a gradual build-up process of a distribution / wave function similarly to, say, Feynman virtual paths making up an exact transition amplitude. That is, this interpretation pretends to no more than conveniently portray virtual events in an amplitude world, and as such is fundamentally different from the real default mechanism, discussed by W. Furry in 1936* [4], *as well as from the general view of Naïve realism.*

And further, since a wave function can be routinely transformed between any pair of representations, this *assignment* is deemed happening in every desirable representation which can be revealed at the measurement stage by properly tuning the equipment.

It feels quite important to avoid potential misunderstandings by commenting as follows.

The key error facilitating a nonlocal interpretation root in an incorrect view of a superposition state and shifting a conceptual focus from the very system to the measuring device. Namely, what is typically assumed is that a superposition state is a sort of *synthetic* state with the system being somehow in all components ***simultaneously***, and it is the measuring device job to *trigger* the transition to a specific component. But that's what is precisely incorrect! According to an understanding of WF as a distribution, superposition stands for a set of potentialities (usually, orthogonal) occurring alternately (not simultaneously!) on only one at a time basis per each ensemble member, and what remains for the measuring device (its detector part) is only to register it.

In more formal words, a superposition state is a set of potentialities each complying with conservation laws and occurring proportionally to its amplitude in a superposition, i.e., the WF is a distribution function in a virtual space of amplitudes. To our knowledge, one of the first (and brilliantly intuitive and appealing!) arguments in support of this fundamental consideration were elaborated as early as in 1932 by J. I. Frenkel [5].

GENESIS AND EVOLUTION OF THE WAVE-PARTICLE DUALITY

So far, the exposition in this note has been resting on a firm ground of well-established facts and accepted views. In this section in reviewing the genesis and evolution of the wave-particle duality (WPD), we outline a hypothetically possible future development of WPD.

First of all, what is invariably observed in all quantum interference experiments with low intensity beams (be it a diffraction on the edge, or on the pinhole or on two narrow slits in the Young paradigmatic scheme) is a gradual emergence of a coherent interference pattern on the screen despite the fact, that beam particles are clearly consecutive and independent. And the standard explanation of this striking effect has been traditionally resting on the concept of wave-particle duality (WPD).

Historically, WPD outgrew from 1) the de Broglie hypothesis associating each micro-particle with the corresponding wave, and 2) experiments, demonstrating diffraction effects in scattering of photons, electrons, etc., i.e., all micro-particles. Conceptually, notions 1) and 2) were solidified in the Copenhagen Interpretation (CI) of WF as a metric of an individual particle, and since then WPD is generally understood as that, all aspects of phenomena in quantum world can be explained either via classical, i.e., particle-like view, or wave-like view, *but not both*. To many, this construct appeared, and still does, as quite a formal, if not somewhat artificial, combination of two incompatibles, but one way or the other, for want of any better alternative it became a main pillar of QM vs CM philosophy. In 1927, Eddington even dubbed this hypothetical hybrid as *wavicle*.

With time, however, it became clear, especially in the context of the Born statistical postulate, that notion 1) above is not quite correct (and CI along with it), and that WFs describe not individual particles, but rather their full congregations: the so-called quantum ensembles.

Independently, but relatedly, with the development of QED at the end 1940s, there began shaping up an interest in what's behind the wave facet of WPD. That was stimulated by a realization that vacuum is not merely an empty space, but rather a special medium, exerting subtle, but experimentally detectable footprint on quantum systems / micro-particles (e.g., Lamb shift, electron-positron pair formation, etc.). As a result, from then on, their began numerous attempts to construct QM as a random motion in fluctuating vacuum fields. Various implementations of this idea would make a way too long list of related works, so we site just a few with references therein [6-9], among many others.

Combining both lines of thoughts, we come to realize, that the apparent diffractive behavior of micro-particles, or, more generally, the wave facet of WPD, might result from a tight coupling between quantum vacuum and micro-particles. Coming out of it is an *interference* pattern, which we have accustomed to take as a *true* interference (as if micro-particles were indeed some real waves or, at least, possessing wave properties, and what constitutes a wave facet of WPD).

At will, extending Feynman ideas of rendering probabilities by means of *virtual* trajectories, one can trace the invisible vacuum hand behind the *interference and diffraction* a bit more quantitatively by invoking the Feynman path integral representation. Namely, each amplitude participating in forming a total probability amplitude can be portrayed as a sum over all virtual paths. In turn, each path can be viewed as furnishing the action minimum for every realization of vacuum fluctuating potential, and, therefore, recasting an interference in terms of vacuum fields. Notably, the same logic applies when connecting vacuum fluctuations with Bohm's quantum potential in his pilot-wave picture. Incidentally, Feynman's formulation of QM is the only successful attempt to date that boils down the *wave*

appearance of quantum amplitudes to interplay of classical trajectories. To recap the hypothesis: the quantum vacuum acts as a huge fluctuating, but stationary and common for all ensemble particles bath, which is what becomes the key source of their apparent wave-like (and coherent!) behavior. That is, a diffractive appearance is nothing but a *mirage*, masquerading the behind-the-scenes-impact of vacuum fluctuations, or, roughly speaking, a mere vacuum impact in disguise. It is instructive in this regard to make a citation from Rumer [1]: …*There does not exist any analogy between the motion of a single particle and a wave. Meanwhile, quite oftentimes one speaks (being incautious) about the wave nature of a single electron, while, in fact, it should be spoken about the wave nature of the whole beam of particles*.

To summarize: needless to say, despite an undeniable appeal of the heuristic picture above and numerous efforts in this regard, it has not been implemented so far in technically solid and experimentally supported manner. It is only the future work that will show to what extent the outlined ideas come true, but until then the WPD will retain its status of the key quantum philosophy concept.

CONCLUSIONS

1. Along with a well acclaimed difference between QM and CM, there exists yet a subtle and often underappreciated mutual similarity helping better understand the transition between them.
2. In the region intermediate between CM and QM (known as quasi-classical) both mechanics apply and quantum probabilities directly follow from classical mechanics.
3. The long-distance correlations between non-interacting particles in QM is the same manifestation of the conservation laws as in CM and, loosely speaking, can be heuristically pictured as such.
4. It is possible that the formation of quantum ensembles is facilitated via an impact of quantum vacuum.

Overall, QM is a particular statistical phenomenology with the starting point postulating a state vector, playing the role of a complex distribution function (WF) in a quantum ensemble of individual events for any desirable representation (e.g., Coordinate/momentum/angular momentum/etc.). Also, at any point in time, by definition, all elementary individual events, and, therefore, entire distribution functions/WFs, are consistent with conservation laws. Any attempt to explain/derive conservation laws in the WF build-up process with a like-WF arguments, effectuates a vicious circle logic and inevitably leads to paradoxes, particular, the quantum non-locality paradox.

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