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Computed Physical Characteristics of the HD 158259 Planetary System: Semi-Major Axis, Orbital Period, and the Triads Resonance of Successive Planets

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ABSTRACT

HD 158259 (or HIP 85268) is one of the members of the main sequence in G0 group stars located approximately at 88 light-years away in the constellation Draco, discussed with respect to the solar system. HD 158259 was discovered by the SOPHIE échelle spectrograph using the radial velocity method. This system includes five confirmed planets orbiting HD 158259, together with one unconfirmed planet. The planets orbit in a near 2/3 (or 3:2) orbital resonance. Starting from the innermost pairing, the period ratios are with the period ratios 1.5757, 1.5146, 1.5296, 1.5128, and 1.4482, respectively, starting from the innermost pairing. Here we theoretically investigate the HD 158259 system for the Semi-major axis, planet's mass, star luminosity, inner, center, outer, and Δ(HZ) habitable zone. We account for radial velocity amplitude, planet density, and Laplace's resonance, theoretically. The existing possibility of the sixth and undetected planet (HD 158259 g) was also investigated. The orbital period and semi-major axis of this planet, computed with 0.047726 years (17.420 days) and 0.135 AU respectively, with R^2 = 0.9964. The radial velocity amplitude of this new and undetected planet was computed to be about 1.625 km/s. The mass of planets in terms of Solar, Jupiter, and Earth have shown direct direct proportional relation with an approximate increase in their semi-major axis. The lowest mass is closest to the star and the highest mass is farthest from the star. We compare the habitability zone to that of NASA, Kopparapu et.al, and the original Kopparapu estimate. The application of the relative mean motion ration (RMMR) for resonance in the triads of successive planets showed that the mode of RMMR is approximately 2/3 (or 3:2) orbital resonance, with the calculated period ratios shown above. we have also calculated the planetary equilibrium temperature (PET) in terms of the size, temperature, Albedo and distance planet to its parent star.

Keywords: New Exoplanet; HD 158259 system; new planets.

INTRODUCTION

Up to now, scientists have classified exoplanets into different categories based on their size and incident stellar flux [1] such as Gas giants, Neptunian, super-Earth, and terrestrial of various sizes, from gas giants larger than Jupiter to small, rocky planets about as big around as Earth or Mars [2, 3]. They can be hot enough to boil metal or locked in a deep freeze. A gas giant is a large planet mostly composed of helium and/or hydrogen. These planets, like Jupiter and Saturn in our solar system, do not have hard surfaces and instead have swirling gases above a solid core. Gas giant exoplanets can be much larger than Jupiter, and much closer to their stars than anything found in our solar system [4]. Super-Earths exoplanets are different from any planets in our solar system. They are more massive than Earth yet lighter than ice giants like Neptune and Uranus and can be made of gas, rock, or a combination of both. They are between twice the size of Earth and up to 10 times its mass. Neptunian exoplanets are similar in size to Neptune or Uranus in our solar system. Neptunian planets typically have hydrogen and helium-dominated atmospheres with cores or rocks and heavier metals. A terrestrial planet, or rocky planet, is a planet that is composed primarily of silicate rocks or metals. According to the definition with IAU the terrestrial planets in our solar system are the inner planets closest to the Sun, i.e. Mercury, Venus, Earth, and Mars. Since they have a solid planetary surface, therefore makes them significantly different from the larger gaseous planets.All terrestrial planets in the Solar System have the same basic structure, such as a central metallic core, (mostly iron) with a surrounding silicate mantle [4].

In a new classification scheme, a planet-size and the relationship between its size and mass appear to be primarily driven by volatile inventory. For example, the atmospheric composition of larger planets is predominantly H2/He, while smaller planets can have a mixture of CH4, CO2, H2O, and NH3. For high-temperature atmospheres, such as hot Jupiter, should have their chemical and spectral features should have primarily determined primarily by equilibrium chemistry. Low-temperature atmospheres will have chemical features determined by photochemistry, but this will be secondary to determining what species are condensing in their atmospheres [1].

In this paper, we investigated the physical properties of five confirmed planets of the HD 158259 (HIP 85268 TIC ID: 188768068, TOI 1462.01) systems as a planetary system in the constellation Dragon that is located in 17:25:24.05529 +52:47:26.4699 with 88.3 light-years (27.1 pc) away from our solar system.

The star’s name and aliases are indicated as: TIC 188768068, 2MASS J17252406+5247263, BD+52 2057, Gaia DR2 1416050859226670848, HD 158259, HIP 85268, IRAS 17242+5249, SAO 30380, TOI-1462, TYC 3888-01886-1, UCAC4 714-055846, WISE J172523.95+524725.9 [5-7].

HD 158259 is a G0 type main-sequence star with a surface temperature of 6050 K and an effective temperature between about 5,300 and 6,000 K. Its apparent magnitude is 6.5 and the absolute magnitude of 4.3. Typically, a G-type main-sequence star often called a yellow dwarf, or G star is a main-sequence star of spectral type G. Such a star has about 0.9 to 1.1 solar mass and is 1.2 times bigger in comparison with the Sun. Like other main-sequence stars, a G-type main-sequence star is converting the element hydrogen to helium in its core by means of nuclear fusion. Besides the Sun, other well-known examples of G-type main-sequence stars include Alpha Centauri A, Tau Ceti, and 51 Pegasi.

Theoretical computation of Semi-major axis, planet's mass and density, stellar luminosity, habitability, radial velocity amplitude, Laplace’s resonance, angular velocity, and equilibrium temperature are the aim of our intention for this work.

Moreover, the existing possibility of an undetected planet as a sixth planet was also considered. For this predicted planet, the values of the orbital period and semi-major axis were computed [3, 8] with 0.047726 (y) and 0.135 (AU) respectively. The coefficient of determination also was achieved at 0.9964.

The radial velocity amplitude of this new and undetected planet was also computed at about 1.625 km/s. The habitability zone values of this system are correspondingly our interesting section with fair comparison to NASA, Kopparapu et.al, and our calculation for the inner, center, and outer regions. The relative mean motion ratio (RMMR) was applied in a theoretical study of the resonance in the triads of successive planets of this system. The result shows that the mode of RMMR is approximately very close to 2/3.

DATA

All data used in this paper were selected from “*The NASA Exoplanet Archive*”[6] and “*The Habitable Exoplanets Catalog*” [2] which are online sources of data and cataloged from PHL for MR relationship. All data collected were updated by end of March 2022. Known stellar parameters of HD 158259 have been shown in Table 1.

Table 1. Known stellar parameters of HD 158259

Parameters	Hara et al. 2020 [9]	TICv8 [10]	Gaia DR2 [11]	Turnbull 2015 [12]	Tokyo Exokyoto [13]	ExoFOP [14]
T_{eff} (K)	---	5801.8900	5928	5834	6050	5801.89
L ($\log_{10}(L_\odot)$)	---	0.21275209	0.21232664	0.243 +0.012-0.013	0.2449	1.63212
P (g/cm^3)	---	0.7253204	---	---	---	---
M ($M_\odot$)	1.08	1.040000	---	1.15	1.080	1.04
R ($R_\odot$)	1.21	1.2644200	1.2105969	1.29	1.2100	1.264420
log g($\log_{10}$(cm/s^2))	---	4.2513200	---	---		4.25132
Sp. T	G0	---	---	G0	G0	G0
γ (km/s)	---	---	13.392442421139847	---	---	---
v sin i (km/s)	2.9	---	---	---	---	---
P_{rot} (days)	20	---	---	---	---	---
Absolute Magnitude	---	---	---	---	4.30	---
Apparent Magnitude	---	---	---	---	6.46	---

All collected data updated by March 2022. Known stellar parameters of HD 158259 have been shown in Table 1.

SEMI-MAJOR AXIS CALCULATION

The most important and essential physical property of exoplanets is the semi-major axis of planets. Here, the calculation of the semi-major axis of the five confirmed exoplanets is done. All data were collected from NASA Exoplanet Archive [4]. Here we compute the semi-major axis of all confirmed five exoplanets using Newton's modification of Kepler's third law for the host stars and orbital period of planets. Where, M_s and M_p are the mass of a star and the exoplanets, and T and a are correspondingly the orbital periods and semi-major axis of planets.

$$(M_s + M_p)T^2 = a^3 \tag{1}$$

Since the mass of planets are very small in comparison to the mass of their host star, therefore equation (1) reduces to its simple form as;

$$M_s T^2 = a^3 \tag{2}$$

or

$$a = \sqrt[3]{M_s T^2} \tag{3}$$

Hence, using the Python code, we have computed the values of the semi-major axes of the planets in AU (Table 1).

Table 2. Computed Semi-major axes via applying Kepler's Third Law for confirmed Exoplanets.

No	Planets Name		Ms	Rs	$M_p(M_\oplus)^*$	R_p	$T(y)$	a (AU)
1	HD 158259 b	(Confirmed)	1.08	1.21	2.23	1.200	0.0060	0.0338
2	HD 158259 c	(Confirmed)	1.08	1.21	5.59	1.574	0.0094	0.0457
3	HD 158259 d	(Confirmed)	1.08	1.21	5.42	1.560	0.0142	0.0603
4	HD 158259 e	(Confirmed)	1.08	1.21	6.08	1.607	0.0218	0.0800
5	HD 158259 f	(Confirmed)	1.08	1.21	6.14	1.611	0.0330	0.1055
6	HD 158259 g	(Not Confirmed yet)	1.08	1.21	6.90	-	0.0477	0.1350

Consequently, the relation between the orbital period and the semi-major axis of planets in terms of the planet’s mass has been shown in Fig 1.

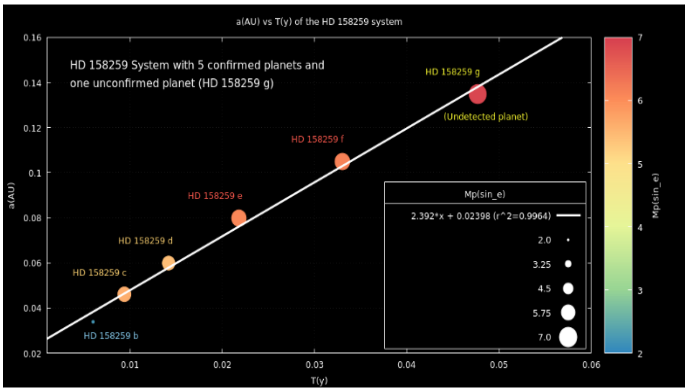

Fig 1. Orbital Period-Semi major axis of HD 158259 Exoplanets with $R^2 = 0.997$

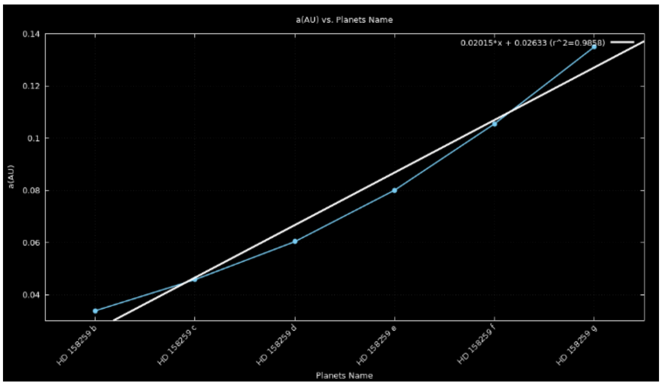

Fig 2. Configuration of the system with the Titius-Bode law with $R^2 = 0.9858$

Fig. 3, show exoplanets of HD 158259 in semi-major axis and mass in color. The first planet shown is in blue with the smallest mass and nearest planet to the primary. This Fig shows that the mass of the planets increases with increasing semi-major and or distance of planets from the star. The mass and semi-major of the unconfirmed planet (HD 158259 g) were measured with $6.9M_\oplus$ and 0.13AU respectively.

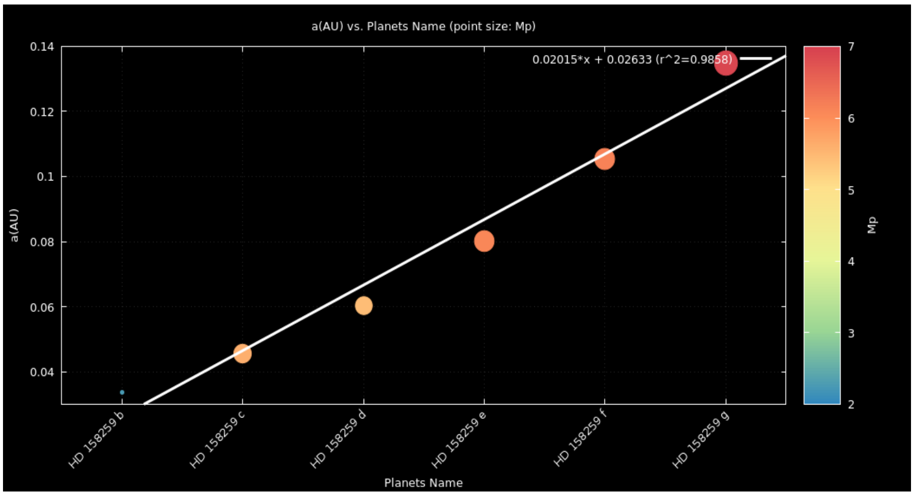

Fig 3. Semi-major axis vs mass of planets in color

MASS CALCULATION

The mass of planets can be computed with using the equation 4. where a , is the semi-major axis of the planets, M_s is the stellar mass of the HD 158259, T as orbital period of the planet around the center of mass of its star and K is the radial velocity amplitude of the planets. We assume the orbit of the planets is circular and not elliptic. Using the conservation of momentum law and some unit conversion, we can find out the mass of the planet M_p in terms of solar mass.

$$M_p = M_s\left(\frac{K\,T}{2\pi a}\right) \tag{4}$$

Consequently, the mass of the planet in terms of solar mass is calculated by the following equation.

$$M_p = (2.1 \times 10^{-4}) M_s\left(\frac{K\,T}{2\pi a}\right) \tag{5}$$

The converted mass of the planets from solar mass to Jupiter and Earth-mass is estimated as [3] in (Table2). Here $M_{p(sol)}$, $M_{p(jup)}$ and $M_{p(Ea)}$ are the mass of planets in terms of Solar, Jupiter and Earth-mass, respectively.

Table 3. Computed mass of exoplanets by equation (5) in terms of Solar, Jupiter, and Earth mass.

No	Planets Name	Rs	Ms	Rp	K(m/s)	Peak Period(d)	T(y)	a(AU)	$M_p(M_\odot)$*	$M_p(M_j)$*	$M_p(M_\oplus)$*	Mass**
1	HD 158259 b	1.21	1.08	0.107000	1.05103	2.178	0.00596	0.034	6.71E-06	0.007	2.23	2.22
2	HD 158259 c	1.21	1.08	0.140360	2.261	3.432	0.00940	0.046	1.68E-05	0.018	5.59	5.60
3	HD 158259 d	1.21	1.08	0.139128	1.9071	5.198	0.01424	0.060	1.63E-05	0.017	5.42	5.41
4	HD 158259 e	1.21	1.08	0.143320	1.857	7.951	0.02178	0.080	1.83E-05	0.019	6.08	6.08
5	HD 158259 f	1.21	1.08	0.143680	1.633	12.028	0.03295	0.105	1.84E-05	0.019	6.14	6.10
6	HD 158259 g *	1.21	1.08	?	** 1.625	17.420	0.04772	0.135	2.08E-05	0.021	6.95	6.91

* Calculated, ** Data from Paper

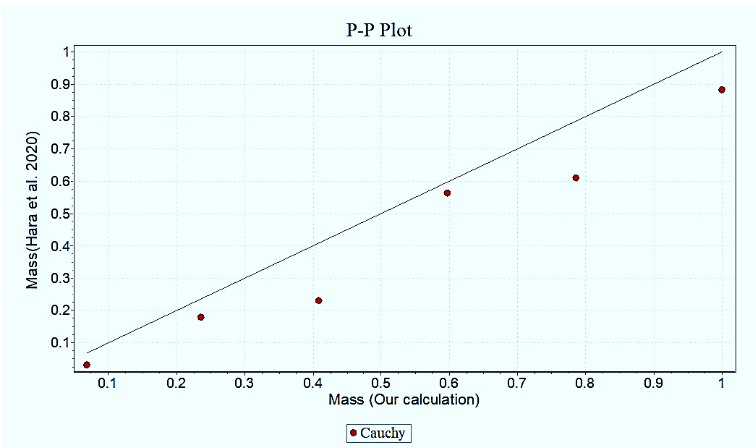

Fig 4. Computed mass of planets. P-P Plot Goodness of Fit with Cauchy Distribution Kolmogorov-Smirnov

PLANET DENSITY

Planet Earth has the highest density of any planet in the Solar System, at 5.514 g/cm3. This is considered the standard by which other planet's densities are measured. This property simply calculated with the mass and radius of the planets. Normally planets are in a different wide range of sizes and masses but those planets made of the same material will have the same density regardless of their size and mass. Since planets are roughly spherical, we calculate the volume of a sphere using the radius.

$$\rho = \frac{M_p}{V} = \frac{M_p}{\left(\frac{4}{3}\pi R_p^3\right)} \tag{6}$$

A huge and quite massive planet can have the same density as a small, low-mass planet if they are made of the same material.

Table 4. Planet density

Planet	Type of planet	M_p (Earth)*	R_p*	ρ (in Earth)
HD 158259 b	Super Earth-Rocky larger than Earth	2.23	1.200	0.309
HD 158259 c	Neptune-like Ice giant planet	5.59	1.574	0.342
HD 158259 d	Neptune-like Ice giant planet	5.42	1.560	0.341
HD 158259 e	Neptune-like Ice giant planet	6.08	1.607	0.350
HD 158259 f	Neptune-like Ice giant planet	6.14	1.611	0.350
HD 158259 g	?	6.91	-	-

* Calculated

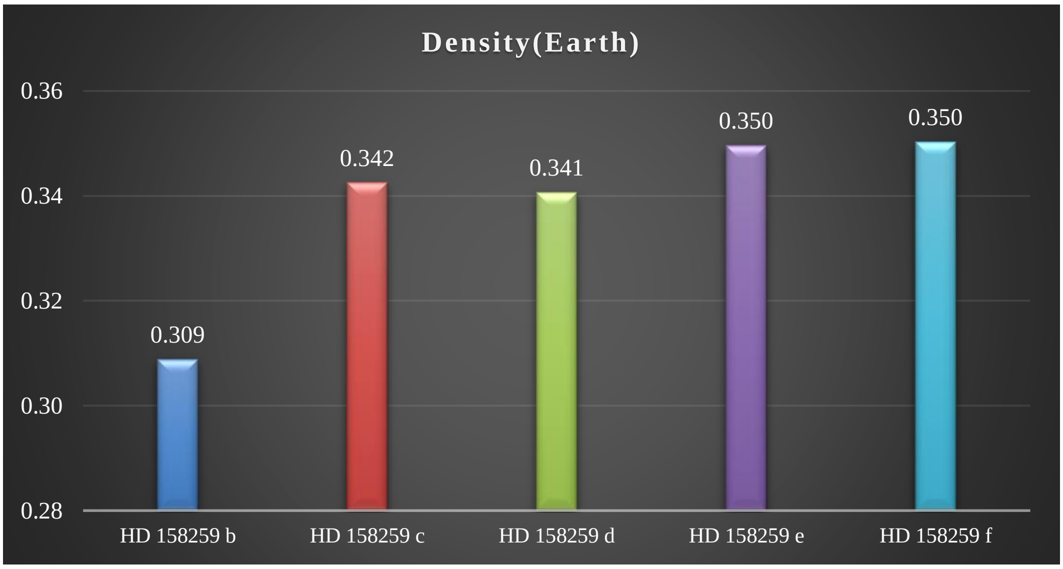

Fig 5. Density of Planets in HD 158259 system.

STELLAR LUMINOSITY OF STAR ($L_\odot$)

The luminosity and flux of stars are important astronomical and physical parameters that can be used to obtain more information and features about stars.

A star's luminosity is the total amount of energy that a star puts out as light each second and if we have a light detector such as an eye, camera, and or telescope then we can measure the light produced by the star.

Therefore, the total amount of energy intercepted by the detector divided by the area of the detector is called Flux and follows the below relation given below.

$$\frac{F_A}{F_B} = \left(\frac{L_A}{L_B}\right)\left(\frac{d_B}{d_A}\right)^2 \tag{7}$$

Here F, L, and d are the flux, luminosity, and distances of stars respectively. However, the aim here is to calculate the luminosity. The luminosity can be determined in two ways. The first way is in terms of radius and effective temperature of the star and the second is using the star's apparent brightness and distance. Here we have computed in terms of radius and effective temperature of the star. The star's radius R_s and the temperature T_s are in solar units.

$$L_s = L_\odot \left(\frac{R_s}{R_\odot}\right)^2 \left(\frac{T_s}{T_\odot}\right)^4 \tag{8}$$

The luminosity values of HD 158259 are recomputed and shown in Table 5 in comparison with different catalogs with equation 8 in terms of the effective temperature and luminosity of the sun. ($T_\odot = 5772\ K$, L= $3.828 \times 10^{28}\ w$).

Table 5. Recomputed values of stellar luminosity for HD 158259 to compare with different catalogs.

	Data from NASA Exoplanet				Computed Values		
Catalogs	R_s [4]	T_{eff} (ºK) [4]	L_s $(log10(L_{\odot}))$ [4]	M_s ($M_{\odot}$) [4]	$T_{eff}/T_{\odot}$	L	L_s $(log10(L_{\odot}))$
Hara et al. 2020	1.21000	-	-	1.080	-	-	-
TICv8	1.26442	5801.89	0.21275209	1.040	1.005178	1.632	0.212755
Gaia DR2	1.21059	5928	0.21232664	-	1.027027	1.631	0.212327
Turnbull 2015	1.29000	5834	0.24300000	1.15	1.010742	1.737	0.239740
Tokyo Exokyoto	1.21000	6050	0.24490000	1.080	1.048164	1.767	0.247287
ExoFOP	1.26442	5801.89	0.21275209	1.04	1.005178	1.632	0.212755

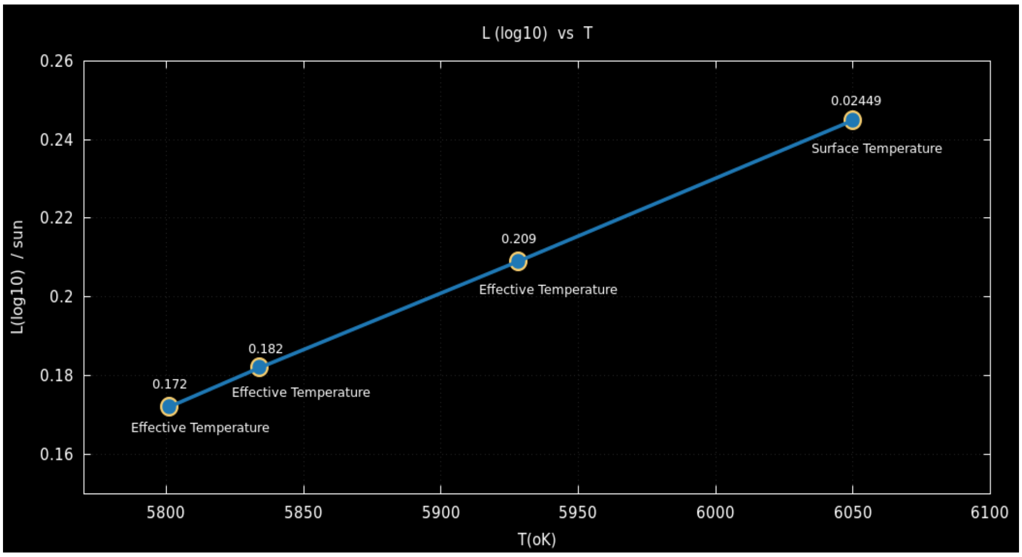

Fig 6. Recomputed values of stellar luminosity vs Temperature for HD 158259.

CALCULATED HABITABILITY

The habitable zone (HZ) is a theoretical and imaginary calculated region surrounding a star throughout which the surface temperatures of any planets present might be conducive to the origin and development of life, as we know it. These regions simply are the range of orbits around a star on which a planetary surface can support liquid water given sufficient atmospheric pressure. For a Sun-like, star (G2), these values are given at 0.75 and 1.77AU. To estimate the HZ of other stars, we neglect the stellar effective temperature and define the center of HZ as 1AU for Earth around the Sun, and scale with stellar luminosity.
According to Equations 9, 10, and 11, and using the luminosity of the star in terms of the luminosity of the sun, it is possible to calculate the inner, center, and outer regions of the Habitability Zone for exoplanets [3, 15].

$$R_{HZ(in)}(AU) = 0.75\sqrt{\frac{L_s}{L_{\odot}}} \tag{9}$$

$$R_{HZ(cent)}(AU) = 1.0\sqrt{\frac{L_s}{L_{\odot}}} \tag{10}$$

$$R_{HZ(out)}(AU) = 1.77\sqrt{\frac{L_s}{L_{\odot}}} \tag{11}$$

$$\Delta(HZ) = R_{HZ(out)} - R_{HZ(in)} \tag{12}$$

Finally, the habitability zone of HD 158259 was calculated for the Inner, Center, Outer, and $\Delta(HZ)$ region. (Table 4).

Table 6. Computed habitable zone of value HD 158259

Method of calculation	Calculated Habitability Zone				Ref
	Hz (Inner)	Hz (center)	Hz (Outer)	$\Delta(HZ)$	http://www.exoplanetkyoto.org/
NASA announcement	0.959	1.326	2.021	1.062	http://www.exoplanetkyoto.org/
Kopparapu et al., 2013	0.978	1.239	2.173	1.195	http://www.exoplanetkyoto.org/
Original Kopparapu	0.985	1.303	2.212	1.227	http://www.exoplanetkyoto.org/
Our Calculation	0.9943	1.3257	2.3465	1.3522	Our calculation

https://www.universeguide.com/star/85268/hip85268#alternativenames

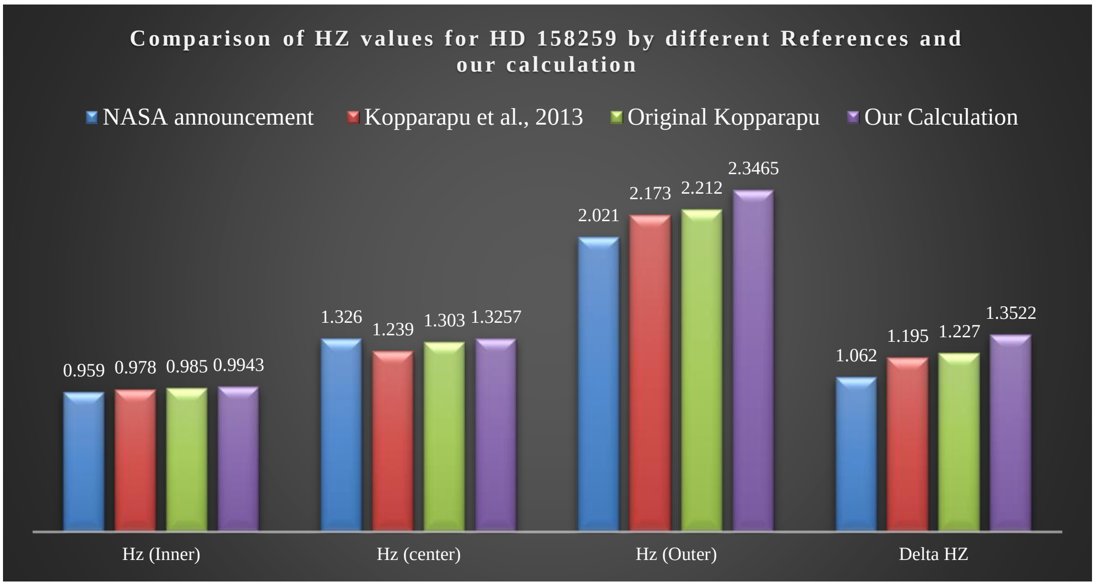

Fig 7. Our calculation of HZ values for HD 158259 and its comparison with different references.

RADIAL VELOCITY AMPLITUDE (RVA)

The radial velocity represents the speed of the object, which moves away (positive sign), and /or approaches (negative sign) from Earth. Here we considered the radial velocity (RVA) amplitude K, of each exoplanet with its estimated mass. The predicted radial velocity amplitude is dependent on the planet orbital period, planet mass, and stellar mass. Using the following expression, the values of K in SI units are shown in Table 7.

$$K = \left(\frac{M_p}{M_s}\right)\left(\frac{a}{T_p}\right)\left(\frac{2\pi}{69.91866}\right) \tag{13}$$

Here M_p and M_s are the mass of the planets and its host respectively, a is the semi-major axis of the planets and Tp is the planet orbital period.

Table 7. Computed Radial velocity amplitude K, of each exoplanet

Planet	$M_s\ (M_\odot)$	M_p	a (AU)	$T(y)$	$K\ (m/s)$
HD 158259 b	1.08	2.22	0.0338	0.006	1.0520
HD 158259 c	1.08	5.60	0.0457	0.0094	2.2784
HD 158259 d	1.08	5.41	0.0603	0.0142	1.8956
HD 158259 e	1.08	6.08	0.0800	0.0218	1.8526
HD 158259 f	1.08	6.14	0.1055	0.0330	1.6270
HD 158259 g	1.08	6.91	0.1350	0.0477	1.6255

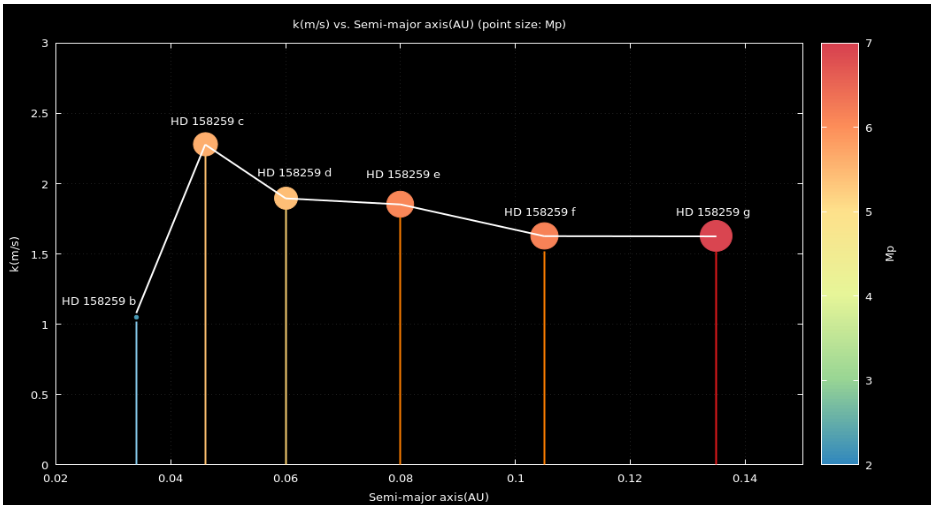

Fig 8. Computed Radial velocity amplitude K, of each exoplanet

RESONANCES

Resonance theory states that if n_1, n_2, n_3 , $(n_i = 2\pi/T_i)$ so that $n_1 > n_2 > n_3$, are mean motions of three planets in circular orbits, then a necessary condition for the frequent occurrence of mirror configuration is given by the following equation [16, 17].

$$\alpha n_1 - (\alpha + \beta)n_2 + \beta n_3 = 0 \tag{14}$$

where α and β are mutually prime positive integers. It follows from equation (14) that in a reference frame rotating with the mean motion of one of three planets, the relative mean motions n_i' of the other two planets are proportionate and we have

$$\frac{n_2'}{n_3'} = \frac{n_2 - n_1}{n_3 - n_1} = \frac{\beta}{\alpha + \beta} \tag{15}$$

For a stable three-body resonance, the relative mean motion ration (RMMR) of $\beta/(\alpha + \beta)$ roughly is equals to 2/3 and therefore the above equation assumes the form

$$\frac{n_2'}{n_3'} = \frac{n_2 - n_1}{n_3 - n_1} = \frac{2}{3} \tag{16}$$

Consequently, this is the Laplace's resonance relation and the three successive orbits obeying this relation can represent stable motion. The application of RMMR to the triads of successive planets in HD 158259 yields the values set out in Table 8. It is clear from the Table that the mode of RMMR is about 2/3. The data of Table 8 show the planets of this system follow the Laplace's resonance relation.

Table 8. Resonance in the triads of successive planets in the HD 158259

No	Triad (Planets Name)	T (day)	a (AU)	$n = 2\pi/T$ (day)	$T_{(i+1)}/T_i$	RMMR $(\beta/(\beta+\alpha))$	
1	HD 158259 b	2.178	0.034	2.883379	$\frac{T_2}{T_1} = 1.5757$		
2	HD 158259 c	3.432	0.046	1.829837		0.628897	$\approx \frac{2}{3}$
3	HD 158259 d	5.198	0.060	1.208157			
2	HD 158259 c	3.432	0.046	1.829837	$\frac{T_3}{T_2} = 1.5146$		
3	HD 158259 d	5.198	0.060	1.208157		0.597770	$\approx \frac{2}{3}$
4	HD 158259 e	7.951	0.080	0.789838	$\frac{T_4}{T_3} = 1.5296$		
3	HD 158259 d	5.198	0.060	1.208157			
4	HD 158259 e	7.951	0.080	0.789838	$\frac{T_5}{T_4} = 1.5128$	0.609758	$\approx \frac{2}{3}$
5	HD 158259 f	12.028	0.105	0.522115			
4	HD 158259 e	7.951	0.080	0.789838			
5	HD 158259 f	12.028	0.105	0.522115	$\frac{T_6}{T_5} = 1.4482$	0.623579	$\approx \frac{2}{3}$
6	HD 158259 g	17.420	0.135	0.360505			

ANGULAR VELOCITY OR ROTATIONAL VELOCITY

The angular velocity or rotational velocity, also known as angular frequency vector [18] is a vector measure of rotation rate, that refers to how fast an object rotates or revolves relative to another point, i.e. how fast the angular position or orientation of an object changes with time. There are two types of angular velocity.

The first is orbital angular velocity that is refers to how fast a point object revolves about a fixed origin, i.e. the time rate of change of its angular position relative to the origin. The second one is spin angular velocity that is refers to how fast a rigid body rotates with respect to its center of rotation and is independent of the choice of origin, in contrast to orbital angular velocity. Here we have estimated orbital angular velocity of planets of HD 158259 in Table 9.

Table 9. Orbital angular velocity of successive planets in the HD 158259

Planets Name	***Ms***	***Rs***	$M_p(M_{\oplus})^*$	***R (planet)***	$Tp(d)$	***a (AU)***	$\omega = 2\pi/T(rad/d)$	$\omega(rad/s) \times 10^{-4}$
HD 158259 b	1.08	1.21	2.23	1.200	2.178	0.0338	2.883	8.01
HD 158259 c	1.08	1.21	5.59	1.574	3.432	0.0457	1.830	5.08
HD 158259 d	1.08	1.21	5.42	1.560	5.198	0.0603	1.208	3.36
HD 158259 e	1.08	1.21	6.08	1.607	7.951	0.0800	0.790	2.19
HD 158259 f	1.08	1.21	6.14	1.611	12.028	0.1055	0.522	1.45
HD 158259 g	1.08	1.21	6.90	-	17.420	0.1350	0.361	1.00

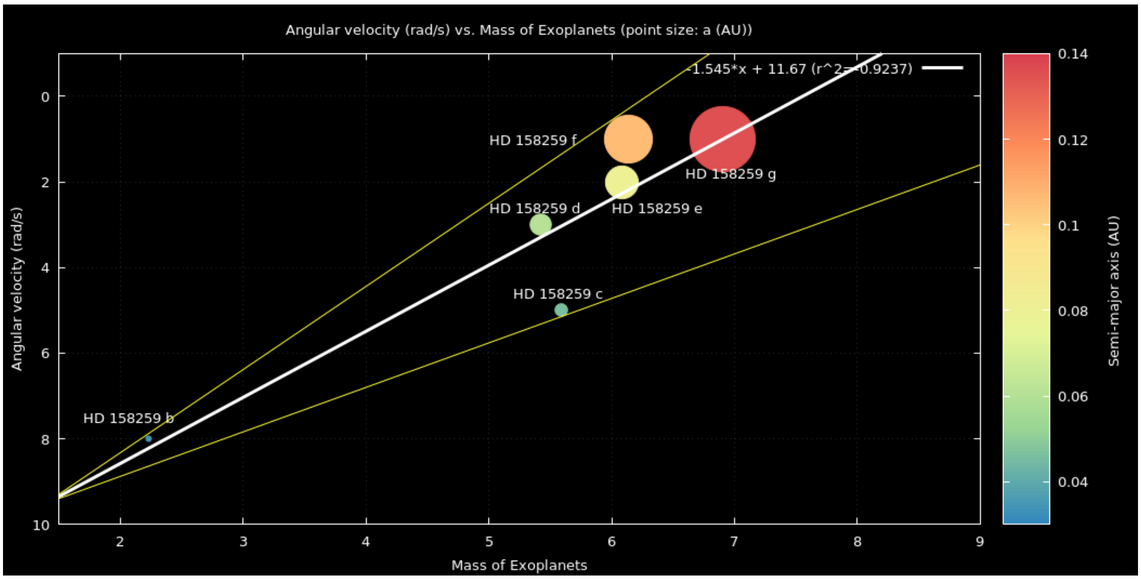

Fig 9. Angular velocity vs mass of exoplanet

Fig 10 shows the relationship between angular velocity and semi-major axis and mass of each planet. The plot indicates that with increasing the semi-major axis of each planet the angular velocity of reduction and the mass of the planet usually increase. However, this is not true for planets c and d contrary to other planets where the above-mentioned assertion stands true.

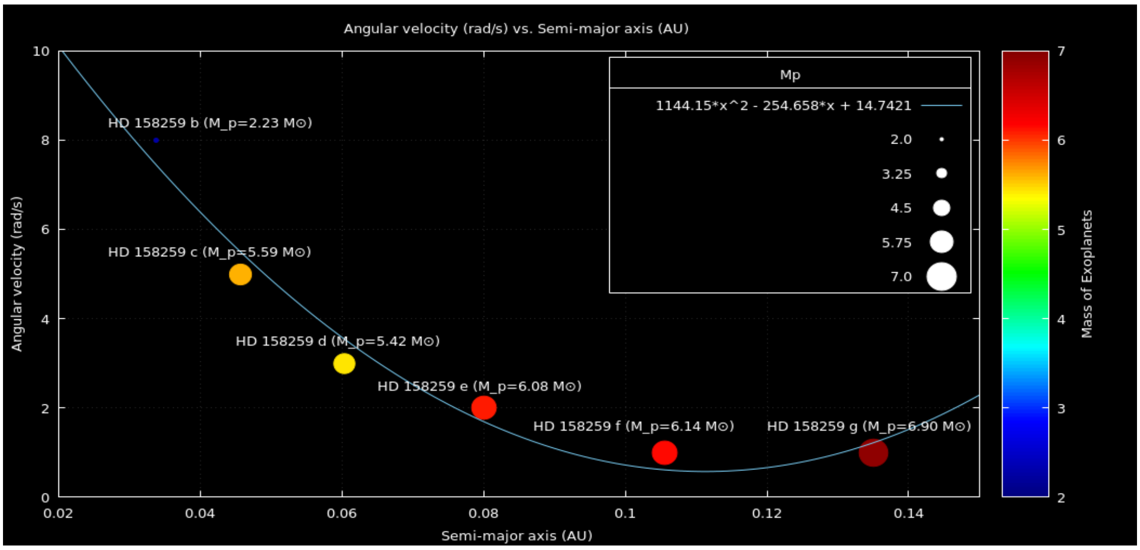

Fig 10. Angular velocity vs semi-major axis of planets

EQUILIBRIUM TEMPERATURES OF PLANETS ALBEDO

The planet's temperature is potentially due to the amount of electromagnetic radiation emitted by its host star since this has the feature of simple black body. Therefore, planet's temperature is not only caused by its atmospheric windows and internal heat sources. In addition, albedo (or reflectivity) value is a fractional amount between 0 and 1 so that albedo value of 0 indicates a surface reflects 0% of incoming solar radiation, while an albedo value of 1 indicates a surface that reflects 100% of incoming solar radiation. The planet's albedo can detect the reflection of shortwave solar radiation in the form of watts per square meter (W/sq m) and simply can be calculated using the following basic equation.

$$Albedo = \frac{\text{Reflected Light}}{\text{Incoming Light}} \tag{17}$$

Now according to the following equation, the luminosity of the host star can be measured with the radius and temperature of the host star.

$$L_{\odot} = 4\pi R_{\odot}^2 \sigma T_{\odot}^4 \tag{18}$$

In the case of extra-solar planets, sometimes the calculated equilibrium temperature is the only available venue for temperature determination, but transit spectroscopy and spectrography over the course of the planet's orbit may offer an independent way for temperature determination in comparison. Therefore, the total amount of energy per unit of time received by the planet is $L_{\odot}\,(R_p/2d)^2$ or $E = (4\pi R_{\odot}^2 \sigma T_{\odot}^4)(R_p/2d)^2$. This is correct if the planet absorbs all the energy, but it is not correct because some of the energy reflected outward. Here we introduce Albedo A as the fraction of energy per unit time that reflected by the planet. Therefore $(1-A)$ is the amount of energy absorbed by the planet. A planet completely covered with snow or ice would have an albedo close to 100%, while a completely dark planet would have an albedo close to zero. Thus, the total amount of energy per unit of time absorbed by the planet is

$$(1-A)E = (1-A)\left[\left(4\pi R_{\odot}^2 \sigma T_{\odot}^4\right)\left(\frac{R_p}{2d}\right)^2\right] \tag{19}$$

Now, if the planet has a temperature T_p then the amount of energy it puts out per unit of time is

$$L_p = \left(4\pi R_p^2\right)\sigma T_p^4 \tag{20}$$

If the planet is in equilibrium, then the amount of energy is absorbing as,

$$\left(4\pi R_p^2\right)\sigma T_p^4 = (1-A)\left[\left(4\pi R_{\odot}^2 \sigma T_{\odot}^4\right)\left(\frac{R_p}{2d}\right)^2\right] \tag{21}$$

or finally

$$T_p = T_{\odot}(1-A)^{1/4}\left(\frac{R_{\odot}}{2d}\right)^{\frac{1}{2}} \tag{22}$$

The above equation shows that planetary equilibrium temperature (PET) is determined by the size, temperature, and distance to its parent star but not by its own size[3]. For example, the value of (PET) for the Earth can be calculated as 260 K^o (-13 C) when, $T_{\odot} = 5780$ K, $A = 40\%$, $R_{\odot} = 7\times10^5$ /km and $d = 1.5\times10^8$ /km = 1 AU. This temperature is below the freezing point of water, and therefore, oceans would be largely frozen and this is not true. Therefore, what is missing here in the calculation is another effect, known as the Greenhouse Effect. The Greenhouse Effect trapping the heat radiation from the Earth's surface, which raises the mean temperature at the Earth's surface to 300 K. Surfaces with a low albedo appear dark whereas surfaces with a high albedo appear bright.
The average albedo of the Earth from the upper atmosphere is 30–35% (about 0.3) [19] because of cloud cover, but widely varies locally across the surface because of different geological and environmental features [20]. Most land areas are in an albedo range of 0.1 to 0.4 [21].

CONCLUSION

In this work, we have analyzed HD 158259 for different physical parameters as below. Orbital period & semi-major axis calculation show the values of 0.0060, 0.0094, 0.0142, 0.0218, 0.0330, 0.0477 (y) and 0.0338, 0.0457, 0.0603, 0.0800, 0.1055 and 0.1350 (AU) for *b, c, d, e, f* and *g* planets respectively. The mass of *b, c, d, e, f* and *g* planets estimated as, 2.23, 5.59, 5.42, 6.08, 6.14 and 6.91in Earth mass unit. Moreover for habitability we could find 0.9943, 1.3257, 2.3465 and 1.3522 for inner, center, outer and Δ(HZ) respectively. Subsequent calculated mass of the planets *b, c, d, e, f,* (except HD-158259 *g*) the density of planets belonging to HD 158259 have been estimated with 0.309, 0.342, 0.341, 0.350 and 0.35 in that order.
The stellar luminosity values of HD 158259 recomputed by equation 8 separately for Hara et al. 2020, TICv8, Gaia DR2, Turnbull 2015, Tokyo Exokyoto, ExoFOP with 0.212755, 0.212327, 0.239740, 0.247287 and 0.212755 in terms of the effective temperature and luminosity of the sun, respectively. According to our calculation the inner, center, outer and Δ(HZ) habitable zone, computed with 0.9943, 1.3257, 2.3465 and 1.3522 as shown in Table 5.
The radial velocity amplitude (*K*) of planets *b, c, d, e, f* and *g* (undetected planet) were computed with 1.0520, 2.2784, 1.8956, 1.8526, 1.6270, and 1.6255 respectively in m/s. According to our dynamical analysis of the system, the planets *b, c, d, e, f,* and predicted planet *g*, have period ratios of $T_2/T_1 = 1.5757$, $T_3/T_2 = 1.5146$, $T_4/T_3 = 1.5296$, $T_5/T_4 = 1.5128$, $T_6/T_5 = 1.4482$. The calculated relative mean motion ration (RMMR) of 0.628897, 0.597770, 0.609758 and 0.623579 roughly is equals to 2/3 (or 3:2). The period ratios are consistent with the distribution of period ratios of planet pairs found in Kepler, which exhibits a peak at 1.52. Finally we have calculated the planetary equilibrium

temperature (PET) which is determined by the temperature, and distance to its parent star and Albedo and not by its own size. The value of (PET) for the Earth is about 260 K^o (-13 C) when, $T_{\odot} = 5780$ K, $A = 40\%$, $R_{\odot} = 7 \times 10^5$ /km and $d = 1.5 \times 10^8$ /km = 1 AU. This temperature is below the freezing point of water.

But the missing parameter here is another effect, known as the Greenhouse Effect.

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