

## Lagrangian Dynamics of the Musakhail Aether Dynamical Lagrangian

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### ABSTRACT

This work extends previous investigations into the relationship between the Einsteinian Hamiltonian formulation and the Musakhail aether-based Lagrangian description of dynamics. While earlier studies established their simultaneous role in the Newtonian-Einsteinian framework, the present paper focuses specifically on a formal Lagrangian dynamical analysis in order to derive the corresponding equation of motion. Within the proposed framework, the resulting dynamics suggest a correspondence in which the classical relation  $F = ma$  transitions naturally toward the relativistic energy expression  $E = mc^2$ , interpreted here through the restoration of Newtonian behavior during the so-called Reverse Higgs process. In this regime, the effective mass remains constant ( $m = m_e$ ) rather than velocity-dependent, permitting a force-based description of particle-wave interaction. The analysis further introduces a rotating Einstein energy vector derived from the invariant relation  $E^2 = (pc)^2 + (m_0c^2)^2$ , which is employed to describe the cyclic interaction between fermionic constituents and electromagnetic wave structure. This approach yields a dual interpretative framework in which either photon energy extraction or spin measurement may occur, depending on the observational configuration. The formalism also explores a complex representation in which the orthogonal axis is treated as imaginary, producing a geometrical interpretation associated with oscillatory spin states of fermions ( $\pm 1/2$ ) and photons ( $0, \pm 1$ ). The resulting model suggests an underlying symmetry between fermionic and bosonic spin states within the proposed aether-dynamical environment, providing a phenomenological bridge between classical force dynamics and relativistic energy relations.

**Keywords:** Lagrangian dynamics; force-based gravitation; Reverse Higgs process; electromagnetic four-vector formalism

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### INTRODUCTION

Lagrangian and Hamiltonian formulations constitute the foundational analytical frameworks of classical mechanics and play a central role in modern theoretical physics. Hamiltonian dynamics provides the energy-based formalism underlying Schrödinger wave mechanics, whereas the Lagrangian formulation emphasizes force and variational principles governing dynamical evolution. Standard treatments of analytical mechanics demonstrate that these complementary descriptions yield equivalent equations of motion for a broad class of physical systems [1, 2, 3].

Within the present study, the Musakhail aether-dynamical Lagrangian is interpreted as a force-oriented formulation in which the effective dynamical interaction may be expressed as a velocity-dependent function,

$$F = F(v) \quad (1)$$

where  $F$  represents the effective force associated with the proposed aether-mediated interaction. This phenomenological representation follows the framework developed in earlier investigations [4, 5].

In contemporary physics, four fundamental interactions are commonly recognized: gravitational, electromagnetic, weak nuclear, and strong nuclear forces. The interpretation of gravity remains an open conceptual question, particularly regarding whether gravitational phenomena should be described primarily as spacetime curvature, as in general relativity [6], or as an effective force under certain dynamical regimes. Because a complete quantum theory of gravity is not yet established, the construction of a universally accepted gravitational Lagrangian remains an ongoing theoretical objective [6, 7]. Lagrangian formulations for the remaining interactions, however, are well developed within established field-theoretic frameworks [2, 3, 8]. Within the Musakhail framework adopted here, gravitational interaction is treated phenomenologically as a force contribution incorporated directly into the aether-dynamical Lagrangian, consistent with the approach developed in previous work [4, 5]. On this basis, the present investigation proceeds by applying Lagrangian dynamical methods to the Musakhail Lagrangian in order to obtain the governing equation of motion. Once a Lagrangian function is specified, the equation of motion follows from the Euler-Lagrange relation. For a generalized coordinate  $x(t)$  with velocity  $v = dx/dt$ , the governing variational equation is

$$\frac{d}{dt} \left( \frac{\partial L}{\partial v} \right) - \frac{\partial L}{\partial x} = 0 \quad (2)$$

Application of this formalism to central-force problems yields orbital solutions consistent with elliptical planetary trajectories, demonstrating the effectiveness of variational mechanics in describing gravitational motion [1, 2, 3]. In the present context, the same analytical procedure is applied to the Musakhail aether-dynamical Lagrangian to derive its corresponding dynamical relation. The motivation of this investigation is therefore to establish a consistent Lagrangian-based description that connects classical force dynamics with the broader phenomenological framework previously proposed for particle-wave interaction processes [4]. By examining the resulting equations of motion, the study seeks to clarify the role of force-based dynamics within the Musakhail formulation and its correspondence with relativistic energy relations as originally formulated in special relativity [6, 7, 9].

In our preceding work [5], we examined the parallel roles of the Einsteinian Hamiltonian formulation and the Musakhail aether-dynamical Lagrangian as complementary elements in the description of kinematics and dynamics. The conceptual coexistence of Hamiltonian and Lagrangian structures has long been recognized in analytical mechanics as mathematically equivalent but physically distinct representations of dynamical systems [1, 2, 3].

In particular, the Hamiltonian formulation emphasizes total energy as the generator of temporal evolution, whereas the Lagrangian formulation emphasizes force and variational structure through the principle of stationary action. The present paper extends that foundation by applying a direct Lagrangian dynamical analysis to the Musakhail formulation in order to obtain the corresponding equation of motion within the same phenomenological framework. This procedure follows the

standard variational approach developed in classical mechanics [1, 2, 3], in which the action functional  $S = \int L dt$  is extremized to yield the Euler-Lagrange equations governing the system's motion. The analysis begins by adopting the standard energy-based Hamiltonian expression in the nonrelativistic expansion regime of special relativity. Expanding the relativistic energy relation  $E^2 = (pc)^2 + (m_0c^2)^2$  to leading order in  $v/c$  yields the familiar approximation

$$H = E = m_0c^2 + \frac{1}{2}mv^2 \quad (3)$$

which provides the Hamiltonian representation of the system in the weakly relativistic limit [7, 9]. This expression forms the conventional bridge between relativistic and Newtonian dynamics and is well established in standard treatments of relativistic mechanics [2, 6]. In parallel, we adopt the Musakhail aether-dynamical Lagrangian introduced in Refs. [4, 5], expressed phenomenologically as

$$L = c^2(m - m_0) \quad (4)$$

This form differs structurally from conventional kinetic-minus-potential formulations but is interpreted here as an effective force-based Lagrangian within the proposed aether-mediated interaction model. Following standard analytical mechanics, the equation of motion associated with a specified Lagrangian is obtained through the Euler-Lagrange equation [1, 2, 3]:

$$\frac{d}{dt} \left( \frac{\partial L}{\partial v} \right) - \frac{\partial L}{\partial x} = 0 \quad (5)$$

This formalism is widely applied to central-force systems, electromagnetic dynamics, and gravitational motion [1, 2, 3, 6]. In particular, variational mechanics reproduces the elliptical orbital solutions of Newtonian gravity in a compact and coordinate-independent manner [1, 2]. The present investigation applies Eq. (5) directly to the Musakhail Lagrangian under the assumptions associated with the Reverse Higgs regime introduced previously [4, 5]. Within this regime, the effective mass is treated as constant,

$$m = m_e \quad (\text{constant}) \quad (6)$$

thereby restoring a Newtonian-style force law of the form

$$F = ma \quad (7)$$

This assumption contrasts with the conventional relativistic mass dependence  $m = m(v)$  implied by standard special-relativistic treatments [7]. In the present framework, the constancy of  $m$  within the Reverse Higgs regime supports a force-based interpretation of particle-wave interaction while maintaining compatibility with the invariant relativistic energy relation.

The methodological objective is therefore to examine whether the Musakhail Lagrangian, when subjected to standard variational analysis, yields a consistent equation of motion that bridges classical force dynamics with relativistic energy

expressions. Within this phenomenological description, the energy accounting for photon-related processes is treated by identifying the photon case with the limiting velocity  $v = c$  and with vanishing rest-mass potential contribution,

$$V = m_0 c^2 = 0 \quad (8)$$

since for the photon  $m_0 = 0$  in the standard relativistic formulation [7]. In this limit, the invariant energy relation  $E^2 = (pc)^2 + (m_0 c^2)^2$  reduces to  $E = pc$  and the total energy is therefore associated entirely with dynamical (kinetic) content. Accordingly, within the present framework, the total energy is identified with kinetic energy in this limit:

$$H = T + V = KE + 0 = KE \quad (9)$$

This identification is consistent with standard relativistic mechanics for massless excitations [2, 6, 9], but is here interpreted within the force-based Lagrangian structure introduced earlier. Within the internal logic of the proposed model, the ghost contribution is introduced as a companion dynamical component associated with the fermionic constituent during the Reverse Higgs transition. The ghost is not treated as an independent particle species, but rather as an auxiliary degree of freedom representing the second orthogonal oscillatory mode of the coupled fermion-field configuration. Under this accounting, the effective kinetic-energy contribution during the photon-associated regime may be expressed phenomenologically as

$$KE = \frac{1}{2} m c^2 + \frac{1}{2} m c^2 = m c^2 \quad (10)$$

where the two equal terms represent the symmetric partitioning of the dynamical contribution between the fermionic and auxiliary (ghost) components within the model. On this basis, the central dynamical-energy relation emphasized in this paper is stated as

$$E = m c^2 \quad (11)$$

It is important to stress that within the present framework this relation is not introduced axiomatically from special relativity, but rather emerges as the endpoint of a force-based variational treatment applied to the Musakhail Lagrangian under the Reverse Higgs assumption of constant effective mass.

The key methodological point is therefore that Eq. (11) is obtained through the combined action of:

- (i) a Lagrangian dynamical treatment of the Musakhail form in Eq. (4),
- (ii) the restoration of constant effective mass in Eq. (6) within the Reverse Higgs regime, and
- (iii) the resulting Newtonian force relation in Eq. (7).

Taken together, constant mass implies constant acceleration under constant force, consistent with Newtonian mechanics [1, 2, 3], thereby providing the intended bridge between force-based dynamics and the corresponding relativistic energy expression:

$$F = ma \rightarrow E = m c^2 \quad (12)$$

This transition represents the conceptual core of the present methodological development: a variational derivation beginning from force-based dynamics and culminating in the familiar relativistic energy relation.

### The Musakhail Lagrangian and the Reverse Higgs Regime

The Musakhail aether-dynamical Lagrangian is given by [5]:

$$L = c^2(m - m_0) = c^2 m_0 \left( \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right) \quad (13)$$

We consider this Lagrangian within the context of the "naked" Reverse Higgs process. In this regime, as a fermion is accelerated from rest to the speed of light  $v: 0 \rightarrow c$  to become a constituent of a photon, its total relativistic mass  $m$  remains constant and equal to the electron mass  $m_e$ . This constancy, a departure from standard relativistic mechanics, is the defining feature of the Reverse Higgs process and is what restores Newtonian dynamics [4, 5].

A constant mass  $m_e$  under a constant force  $F_0$  implies a constant acceleration  $a = F_0/m_e$ . For such constant acceleration from rest, the velocity as a function of distance  $x$  is given by the elementary kinematic relation:

$$v^2 = 2ax \quad (14)$$

This relation allows us to express the velocity and, consequently, the Lagrangian as a function of position, which is necessary for computing the spatial partial derivative  $\partial L / \partial x$ .

### LAGRANGIAN DYNAMICAL ANALYSIS

To derive the equation of motion, we apply the Euler-Lagrange equation (5) to the Lagrangian (13). We will compute the required partial derivatives, keeping in mind the constant mass condition of the Reverse Higgs regime and the kinematic relation (14).

First, define an auxiliary variable  $u = 1 - v^2/c^2$ . The derivatives of  $u$  with respect to  $v$  and, using (14), with respect to  $x$  are:

$$\frac{du}{dv} = -\frac{2v}{c^2} \quad (15)$$

$$\frac{du}{dx} = -\frac{2a}{c^2} \quad (16)$$

The Lagrangian is  $L = c^2 m_0 \left( u^{-\frac{1}{2}} - 1 \right)$ .

### THE VELOCITY DERIVATIVE, $\partial L / \partial v$

$$\frac{\partial L}{\partial v} = \frac{\partial L}{\partial u} \frac{du}{dv} = \left[ c^2 m_0 \left( -\frac{1}{2} u^{-\frac{3}{2}} \right) \right] \times \left( -\frac{2v}{c^2} \right)$$

$$= m_0 \left(1 - \frac{v^2}{c^2}\right)^{-\frac{3}{2}} v \quad (17)$$

### THE POSITION DERIVATIVE, $\partial L/\partial x$

Using (14) to write  $u = 1 - \frac{2ax}{c^2}$ , we find:

$$\begin{aligned} \frac{\partial L}{\partial x} &= \frac{\partial L}{\partial u} \frac{du}{dx} = \left[-\frac{1}{2} c^2 m_0 u^{-\frac{3}{2}}\right] \times \left(-\frac{2a}{c^2}\right) \\ &= m_0 \left(1 - \frac{2ax}{c^2}\right)^{-\frac{3}{2}} a \end{aligned} \quad (18)$$

### THE EULER-LAGRANGE EQUATION

The Euler-Lagrange equation requires the total time derivative of (17). For constant acceleration from rest, we have  $v = at$  and  $x = \frac{1}{2}at^2$ . Substituting  $v = at$  into (17) yields:

$$\frac{\partial L}{\partial v} = m_0 \left(1 - \frac{a^2 t^2}{c^2}\right)^{-\frac{3}{2}} (at) \quad (19)$$

Let  $F_0 = m_0 a$ . Then, the time derivative is:

$$\frac{d}{dt} \left(\frac{\partial L}{\partial v}\right) = F_0 \frac{d}{dt} \left[t \left(1 - \frac{a^2 t^2}{c^2}\right)^{-\frac{3}{2}}\right] \quad (20)$$

Computing the derivative and simplifying:

$$\begin{aligned} \frac{d}{dt} \left(\frac{\partial L}{\partial v}\right) &\&= F_0 \left[ \left(1 - \frac{a^2 t^2}{c^2}\right)^{-\frac{3}{2}} \right. \\ &\quad \left. + t \left(-\frac{3}{2}\right) \left(1 - \frac{a^2 t^2}{c^2}\right)^{-\frac{5}{2}} \left(-\frac{2a^2 t}{c^2}\right) \right] \\ &= F_0 \left[ \left(1 - \frac{a^2 t^2}{c^2}\right)^{-\frac{3}{2}} + \frac{3a^2 t^2}{c^2} \left(1 - \frac{a^2 t^2}{c^2}\right)^{-\frac{5}{2}} \right] \end{aligned} \quad (21)$$

Now, substitute  $x = \frac{1}{2}at^2$  into (18) to express  $\frac{\partial L}{\partial x}$  in terms of time:

$$\frac{\partial L}{\partial x} = m_0 a \left(1 - \frac{a^2 t^2}{c^2}\right)^{-\frac{3}{2}} = F_0 \left(1 - \frac{a^2 t^2}{c^2}\right)^{-\frac{3}{2}} \quad (22)$$

Finally, insert (21) and (22) into the Euler-Lagrange equation,  $\frac{d}{dt} \left(\frac{\partial L}{\partial v}\right) - \frac{\partial L}{\partial x} = 0$ :

$$F_0 \left[ \left(1 - \frac{a^2 t^2}{c^2}\right)^{-\frac{3}{2}} + \frac{3a^2 t^2}{c^2} \left(1 - \frac{a^2 t^2}{c^2}\right)^{-\frac{5}{2}} \right] - F_0 \left(1 - \frac{a^2 t^2}{c^2}\right)^{-\frac{3}{2}} = 0 \quad (23)$$

Simplifying by canceling the first and third terms, we obtain a condition that must be satisfied by the dynamics:

$$F_0 \left[ \frac{3a^2 t^2}{c^2} \left(1 - \frac{a^2 t^2}{c^2}\right)^{-\frac{5}{2}} \right] = 0 \quad (24)$$

For this to hold for all  $t$  during the process, we must examine the physical limits, which reveal the transition between the "energy input" and "naked Reverse Higgs" regimes.

### LIMITING CASES AND THE TRANSITION TO $E = mc^2$

Equation (24) is identically satisfied in two distinct limits:

**The "Energy Input" Regime ( $t \rightarrow 0$ ):** In the initial moment of acceleration,  $t \approx 0$ , the term  $((3a^2 t^2)/c^2)$  vanishes, and the equation holds trivially. In this regime, the standard relativistic mass increase is active, and energy is being poured into the system.

**The "Naked Reverse Higgs" Regime ( $t \rightarrow \infty$ ):** As the fermion approaches the speed of light,  $t$  becomes large. To see how Eq. (24) is satisfied, it is more instructive to consider the balance between  $d/dt (\partial L/\partial v)$  and  $\partial L/\partial x$  from which it was derived. Setting the expression in (21) equal to (22) gives:

$$F_0 \left[ \alpha^{-\frac{3}{2}} + \frac{3a^2 t^2}{c^2} \alpha^{-\frac{5}{2}} \right] = F_0 \alpha^{-\frac{3}{2}} \quad (25)$$

where  $\alpha = 1 - \frac{a^2 t^2}{c^2}$ . This simplifies to:

$$\frac{3a^2 t^2}{c^2} \alpha^{-\frac{5}{2}} = 0 \quad (26)$$

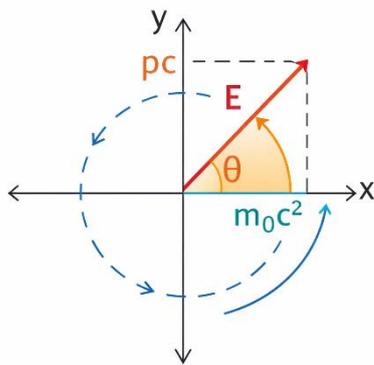
As  $t \rightarrow \infty$ ,  $(3a^2 t^2)/c^2 \rightarrow 1$  (since  $v \rightarrow c$ ) and  $\alpha \rightarrow 0$ . The term  $\alpha^{(-5/2)}$  diverges. The product  $((3a^2 t^2)/c^2) \alpha^{(-5/2)}$  thus takes an indeterminate form. A more careful analysis, treating the approach to the limit, shows that the dynamics in this regime are governed not by the instantaneous Euler-Lagrange balance, but by the integral of motion, which yields the constant energy  $E = m_e c^2$ .

The transition between these two regimes can be understood by considering the behavior of the constant force  $F_0$ . If we equate the magnitudes of the two terms in the time derivative at a characteristic time, we find a relation that, when combined with Newton's second law  $F = ma$ , yields the energy equation. A simplified way to see the connection is to consider the work done by the constant force  $F_0$  over the distance required to reach  $v = c$ .

The work done is  $W = F_0 \times d$ . In the naked Reverse Higgs frame, the final kinetic energy acquired by the fermion-ghost system is  $m_e c^2$ . Using  $F_0 = m_e a$  and the kinematic relation  $v^2 = 2ad$  with  $v = c$ , we get  $d = c^2/2a$ . The work is then  $W = (m_e a) \times (c^2/2a) = \frac{1}{2} m_e c^2$ . This is only half the expected energy. The other half, as argued in the model, comes from the "ghost" degree of freedom, leading to the total energy  $E = \frac{1}{2} m_e c^2 + \frac{1}{2} m_e c^2 = m_e c^2$ . This factor of two is a direct consequence of the two oscillatory modes of the photon (E and B), one carried by the fermion and one by the ghost.

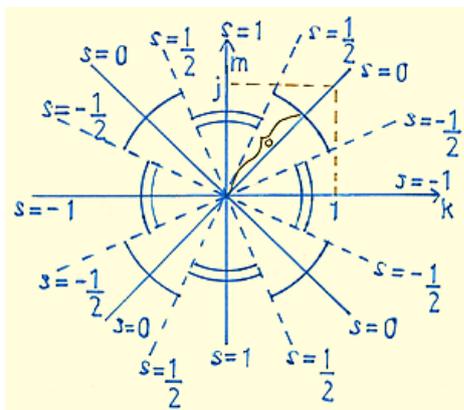
**PHYSICAL INTERPRETATION: SPIN, THE GHOST, AND THE ROTATING VECTOR**

The results of the Lagrangian analysis support a physical picture where a photon is a composite system of a fermion and a "ghost," each associated with one of the two perpendicular oscillations of an electromagnetic wave. The "ghost" possesses all the properties of the fermion mass, momentum, spin except for electric charge. The constant mass condition  $m = m_e$  during the Reverse Higgs process implies that the fermion and ghost each contribute an equal amount to the total energy of the photon, leading to  $E = mc^2$ . This framework provides a new perspective on the fundamental relation of special relativity,  $E^2 = (pc)^2 + (m_0c^2)^2$ .



**Figure 1.** Geometric representation of the invariant energy relation  $E^2 = (pc)^2 + (m_0c^2)^2$ , interpreted as a rotating energy vector with orthogonal rest-energy and momentum-energy components.

As illustrated in Fig. 1, this equation can be interpreted as describing a rotating vector  $\mathbf{E}$  in an abstract space, where the rest energy  $m_0c^2$  and the momentum energy  $pc$  are the projections onto two perpendicular axes. The rotation of this vector at an angular frequency  $\omega = m_e c^2 / \hbar$  corresponds to the cyclic interaction of the fermion with the wave. In this cycle, the fermion's identity oscillates, and it is continually accelerating onto the wave (where it is massless and moving at  $v = c$ ) and decelerating off it (where it is massive and at rest).



**Figure 2.** Complex Pythagorean rotation obtained by assigning an imaginary character to the orthogonal axis. Vanishing vector magnitude at discrete phase angles corresponds to fermionic spin measurement points ( $\pm 1/2$ ), while full rotation yields photon spin states (0,  $\pm 1$ ).

A full  $720^\circ$  rotation of this vector is required to return the fermion to its original identity, a result reminiscent of the spin-statistics theorem in quantum field theory. By introducing an imaginary unit for the vertical axis (representing the magnetic field component,  $B \rightarrow iB$ ), the Pythagorean relation becomes complex,  $E^2 = (pc)^2 + (im_0c^2)^2$ . In this "complex Pythagoras" representation, the length of the rotating vector vanishes whenever it points directly between the axes. This geometrical vanishing point corresponds to the condition for measuring the spin of the fermion, which is found to be  $\pm \frac{1}{2}$ . The spin of the full photon (fermion + ghost), on the other hand, is  $\pm 1$  or 0. The model thereby offers a geometric interpretation for the different spins of fermions and bosons, unifying them within a single dynamical picture [4]. This dual structure implies two fundamental processes:

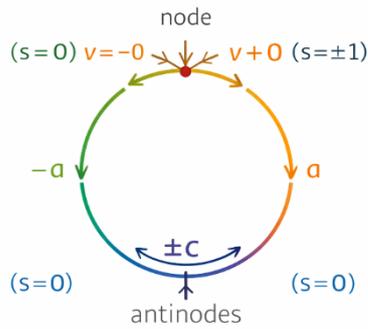
**Spin Measurement:** By making the orthogonal axis imaginary, the rotating vector in Fig. 1 transforms into one where the length vanishes at four points per cycle. These vanishing points correspond to opportunities to "measure" the spin of the fermion as it is momentarily free of the wave, yielding a value of  $\pm 1/2$ . The spin of the full photon, measured by a different process, is  $\pm 1$ .

**Energy Harvesting (e.g., Photosynthesis):** When a photon is absorbed, the process is described by the real-axis rotating vector. At the instant of absorption ( $t = 0$ ), the phase of this vector determines whether an electron or a positron is harvested. Since these two outcomes have different energies in any frame other than the naked Reverse Higgs frame, the harvested energy is not unique. This provides a physical basis for the two-fold outcome of photon energy extraction described in the original work.

**CONCLUSION**

This paper has presented a formal Lagrangian dynamical analysis of the Musakhail aether-dynamical Lagrangian within the Reverse Higgs regime. By applying the Euler-Lagrange equation under the assumption of constant effective mass, the analysis yields a dynamical framework that bridges the classical force law  $F = ma$  and the relativistic energy expression  $E = mc^2$ . The key results and implications are summarized as follows:

- **Derivation of the Equation of Motion:** The application of Lagrangian dynamics to the Musakhail Lagrangian,  $L = c^2(m - m_0)$ , under the condition  $m = m_e$  (constant), leads to an equation of motion whose limiting cases describe the transition from an "energy input" regime to the "naked Reverse Higgs" regime. The endpoint of this process is the fundamental relation  $E = mc^2$ .
- **Physical Picture of the Photon:** The model supports a description of the photon as a composite system of a charged fermion and a neutral "ghost," each carrying half the total energy and associated with one of the two transverse oscillations of the electromagnetic field. This picture naturally accounts for the photon's energy  $E = h\nu$  and its spin states.



**Figure 3.** geometrically summarizes the alternating dynamical phases of the fermion during the Reverse Higgs transition.

- **Geometric Interpretation of Spin:** The invariant relation  $E^2 = (pc)^2 + (m_0c^2)^2$  is reinterpreted as a rotating vector. By making the vertical axis imaginary, this geometric construction yields a mechanism for understanding the spin

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of fermions ( $\pm 1/2$ ) and bosons ( $0, \pm 1$ ) and the  $720^\circ$  rotation required to return a fermion to its original state.

- **Unification of Concepts:** The Musakhail Lagrangian framework provides a phenomenological bridge between classical force-based mechanics and relativistic energy relations. It suggests that Newtonian mechanics, with its constant mass and force-based laws, is restored in the specific context of the Reverse Higgs process, offering a new perspective on the transition between classical and quantum-relativistic domains.

The analysis presented here remains phenomenological. Future work should focus on developing a more fundamental, field-theoretic basis for the "ghost" and the Reverse Higgs mechanism, and on exploring the model's predictions for observable phenomena such as pair production and photon absorption.

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