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An Exploratory Framework for Gravitation and Electrodynamics: A Lagrangian-Hamiltonian Perspective

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ABSTRACT

This paper presents an exploratory, force-based framework for gravitation and electrodynamics, motivated by a correspondence between Musakhail's aether dynamics and Einsteinian special relativity. These two perspectives are interpreted through the lens of Lagrangian-Hamiltonian duality, wherein force-based formulations (Lagrangian) and energy-based formulations (Hamiltonian) are treated as complementary descriptions of underlying physical dynamics. The aims of this work are threefold. First, to develop a force-based interpretation of gravitational interactions by examining Musakhail's force relation, $F = c^2(m - m_0)$ in parallel with the relativistic energy expression, $E^2 = (pc)^2 + (m_0c^2)^2$ highlighting their dual structure. Second, to introduce and analyze two exploratory electromagnetic four-vectors $(J \cdot E, E \times B)$ and $(\hbar\omega, v \times B)$ employing an extremization principle as a heuristic tool for investigating structural analogies between dissipation, Poynting flux, and Lorentz-force dynamics. Third, to explore a minimal-scale electro-gravitational correspondence through helical flux-tube geometries and a constant-mass acceleration mechanism, suggesting possible shared features between fermionic transport and electromagnetic field configurations. The extremization procedure, in which the scalar component of a four-vector is set equal to the magnitude of its vector component, is applied heuristically to reveal formal parallels rather than to derive rigorous field equations. Within this phenomenological model, gravitational interactions are considered as collective nuclear-scale force processes, while electromagnetic energy transport is examined through Lorentz-force cancellation and Poynting-flow relations. The helical flux-tube structures provide a unifying geometric motif, with effective tension identified with Newtonian gravitational force. This work is intended as a conceptual and phenomenological exploration rather than a replacement for established relativistic field theories. Its physical relevance depends on further mathematical development and empirical validation. Several qualitative, testable consequences are outlined to motivate future theoretical refinement and experimental assessment.

Keywords: Lagrangian-Hamiltonian duality; force-based gravitation; electromagnetic four-vectors; helical flux-tube dynamics

INTRODUCTION

The present work examines a conceptual correspondence between Einsteinian special relativity and Musakhail's aether dynamics, interpreted here as complementary descriptions of energy and force, respectively. Within this framework, special relativity provides an energy-based formalism, while Musakhail's approach emphasizes force-mediated dynamics [1]. These two perspectives are treated as jointly informative in constructing a phenomenological model of gravitational and electromagnetic interactions. Acceleration of a massive body to velocity v necessarily involves the cumulative action of force. In this context, relativistic energy may be expressed as

$$E^2 = (pc)^2 + (m_0c^2)^2 \quad (1)$$

This energy-based description corresponds to a Hamiltonian formulation,

$$H = T + V \quad (2)$$

where T represents kinetic energy and V represents potential energy [2]. In classical mechanics, kinetic energy is given by

$$T = \frac{1}{2}mv^2 \quad (3)$$

In parallel, Musakhail's force relation may be written as

$$F(v) = c^2(m - m_0) \quad (4)$$

which is associated with a Lagrangian form,

$$L = T - V \quad (5)$$

This juxtaposition highlights an energy-force duality that motivates the present analysis [1]. The discussion is framed primarily in terms of special relativity rather than general relativity. While general relativity has demonstrated substantial empirical success, particularly in describing photon deflection by massive bodies [3], the present study explores whether macroscopic gravitational interactions such as planetary motion may alternatively be modeled using effective force-based dynamics [4,5].

From this perspective, spacetime curvature and Newtonian gravitational force are regarded as experimentally indistinguishable in certain regimes. Introducing a finite interaction time between gravitational source and field, determined by graviton propagation, leads to orbital corrections analogous to observed perihelion precession. It is proposed that graviton velocity scales with interaction distance, yielding a constant interaction time and introducing an effective rotational contribution to orbital dynamics [6].

Within Musakhail's framework, gravity is treated explicitly as a force arising from interactions between atomic nuclei. Planetary-scale gravitation is viewed as the collective manifestation of these microscopic processes. Accordingly, gravitational dynamics involving macroscopic bodies are considered fundamentally Newtonian in character, while relativistic curvature effects are regarded as becoming significant primarily for massless or near-massless particles such as photons and leptons [1].

This distinction motivates a reassessment of the applicability of geometric spacetime curvature to nuclear-scale gravitational interactions. The present model suggests that atomic nuclei interact via graviton exchange rather than curvature-mediated mechanisms [7,8]. From this standpoint, challenges in reconciling quantum mechanics with general relativity may arise from extending geometric formulations into regimes dominated by force-mediated particle dynamics [6,9].

The framework further proposes that nuclei support flux-tube structures analogous to electromagnetic flux tubes, with graviton transport occurring along helical field configurations. Drawing on analogies with terrestrial and solar electromagnetic flux systems, it is suggested that fermionic transport along such helices provides a mechanism for gravitational interaction [10,11]. Observations of solar flares and coronal loops demonstrate that helically twisted flux tubes are ubiquitous in magnetized plasmas, with these structures exhibiting confinement and transport properties that may have gravitational analogues [10,12].

These helical structures are associated with effective tension, identified here with Newtonian gravitational force. The resulting configuration provides a phenomenological model in which gravitational attraction emerges from field-mediated transport processes, paralleling mechanisms observed in electromagnetic systems [1,12].

The present study develops this conceptual framework through a Lagrangian-Hamiltonian correspondence linking Musakhail's force-based dynamics with relativistic energy formulations. Exploratory four-vector constructions and an extremization principle are introduced to examine internal consistency and establish connections between propagation vectors and amplitude vectors. Analogies with Lorentz-force dynamics are employed to motivate the formalism [2,4].

The work presented here is intended as a conceptual and phenomenological investigation rather than a replacement for established relativistic field theories. Limitations of the approach are acknowledged, and further mathematical development and empirical analysis are required to assess its physical applicability. Throughout this work, emphasis is placed on phenomenological consistency rather than formal completeness [6,13]. The specific aims of this work are:

(1) To develop a force-based interpretation of gravitational interactions by revisiting Musakhail's aether dynamics and clarifying its correspondence with Einsteinian special relativity through Lagrangian-Hamiltonian duality.

(2) To introduce and analyze two electromagnetic four-vectors, $(J \cdot E, E \times B)$ and $(\hbar\omega, v \times B)$, together with their extremization properties, establishing connections between Lorentz-force balance, Poynting-flow relations, and helical flux-tube dynamics.

(3) To explore a minimal-scale electro-gravitational correspondence through fermionic helices and a constant-mass acceleration mechanism, suggesting a shared Lagrangian structure between electromagnetic and gravitational interactions.

LITERATURE REVIEW AND METHODOLOGY: FOUR-VECTOR ANALYSIS

Extremization of Four-Vectors

The principal methodological tool employed in this study is the extremization of four-vectors. This approach is used to investigate the internal consistency of the proposed framework, wherein Musakhail's aether dynamics and Einsteinian special relativity are treated as complementary processes operating at the foundation of physical dynamics [1].

Three four-vectors are considered, two of which are introduced within the present work. Their scalar and vector components are examined under an extremization condition in which the scalar component is set equal to the modulus of the vector component.

This extremization procedure is applied heuristically and does not necessarily preserve dimensional consistency. It is adopted here as a phenomenological constraint intended to reveal structural correspondences between force-based and energy-based formulations [2,4]. Following earlier analysis of the magnetic vector potential [5], the electromagnetic four-vector is written as

$$(\varphi, A) \tag{6}$$

where φ denotes the electric scalar potential and A the magnetic vector potential [6,7]. This provides the starting point for extending extremization techniques to additional four-vector constructions. The spacetime four-vector

$$(ct, x) \tag{7}$$

is taken to play a central role in describing helically structured electromagnetic flux-tube processes, such as those observed in solar flares and terrestrial electrical circuits [10,12].

Electromagnetic Four-Vectors and Flux-Tube Structure

A new electromagnetic four-vector is introduced.

$$(J \cdot E, E \times B) \tag{8}$$

where J denotes internal axial current density and E the corresponding axial electric field. These quantities are related through $J = \sigma E$ with σ representing conductivity. This formulation allows magnetic field lines to be treated as two-dimensional structures in solar-flare processes, where thick helically arranged fields, B acquire azimuthal components associated with vector potential A [10]. Within such flux tubes, the axial magnetic field component B_{ax} assumes the role of effective current density. This leads to the electromagnetic duality

$$B = \nabla \times A \tag{9}$$

$$\mu J = \nabla \times B \tag{10}$$

These relations provide a basis for modeling flux-tube stability and internal transport [6,8]. Non-incremental behavior of the spacetime four-vector is examined in connection with helicity, a characteristic feature of electromagnetic flux structures, particularly in solar flares and terrestrial circuits [10,12].

Additional Four-Vector Construction

An additional four-vector is introduced to describe electromagnetic helices.

$$(\hbar\omega, v \times B) \tag{11}$$

This four-vector is used to analyze massive electromagnetic helices in which fermions propagate helically at velocities $v < c$, in contrast to massless electromagnetic structures such as solar flare flux tubes and terrestrial circuits where the relevant propagation speed is c [1,9]. This construction is applied to electron-positron dynamics and electromagnetic transport processes [5,11].

Relation to Lorentz Force and Extremization Conditions

An equivalence is established between the extremization of the four-vector $(\hbar\omega, v \times B)$ and the condition of zero Lorentz force for electromagnetic propagation. The Lorentz force is written as

$$F_L = q(E + v \times B) \tag{12}$$

Setting $F_L = 0$ yields

$$v \times B = -E \tag{13}$$

This relation is paired with the extremization condition

$$\hbar\omega = |v \times B| \tag{14}$$

Together, these expressions establish a correspondence between propagation dynamics and four-vector extremization [4,7]. The spacetime four-vector, the dissipation four-vector $(J \cdot E, E \times B)$, and the Lorentz-force condition form a conceptual triality linking energy transport, force balance, and electromagnetic helicity. Within this framework, the quantity ħω is interpreted as the energetic counterpart of the electromagnetic field amplitude, with the sign convention allowing for negative-frequency solutions [2,6].

Four-Vector of Central Forces

We consider two newly proposed electromagnetic four-vectors: $(J \cdot E, E \times B)$ and $(\hbar\omega, v \times B)$. These constructions motivate a classification of fermionic states based on relative phase, mass-energy sign, and oscillatory behavior [1,5]. Within this framework, fermionic existence may be organized according to whether electrons and positrons are in-phase or out-of-phase, whether electromagnetic waves possess ghost components, and whether mass-energy contributions share the same sign. Four principal fermionic states are identified: the massive fermion; the weak-strong gauge boson (massless, single oscillation); the radiational fermion (massless, $v = c$, dual oscillation); and massless atomic states, including orbital atomic configurations (Schrödinger orbits) as well as helical propagation along electromagnetic flux-tube surfaces [9,11]. Dirac's formulation predicted the existence of the positron, which in this model is interpreted as possessing negative mass-energy [8]. Electron-positron pair creation from photonic annihilation is described as a phase-separated process, with the two particles differing physically by a relative phase of λ/4. Extension from this "Dirac creation" to other fermionic configurations allows trade-offs between mass-energy sign and phase separation.

In radiative processes, electronic and positronic photons are no longer constrained to opposite mass-energy signs; instead, they share the same sign, producing net propagation of mass-energy in a preferred direction characterized by the Poynting vector,

$$P = \left(\frac{1}{\mu}\right)(E \times B) \tag{15}$$

For the weak-strong gauge boson, the fermion pair e^+/e^- loses its dual oscillatory character, leaving only what is described here as a potential ghost [1]. In this regime, fermions propagate in opposite directions. Similar behavior is observed for massless helical states. These considerations suggest that phase relationships alone may not fully determine fermionic behavior in such configurations, indicating the need for further investigation [5,12].

Additional observations are drawn from optical processes. When light is absorbed by material systems for example, in the interaction of radiation with colored surfaces reemission may occur at approximately half the initial frequency. A similar effect is noted in the photoelectric process. Because the upper limit of human visual perception is close to twice the lower limit, such re-emitted radiation generally falls outside observable ranges [4].

INTERNAL AETHER OSCILLATION: TWO NEW ELECTROMAGNETIC FOUR-VECTORS

Following our investigation of the spacetime four-vector, we now introduce two new electromagnetic four-vectors obtained via extremization. These constructions emerge from considering internal dynamics within electromagnetic flux tubes and their relation to energy propagation [1,5]. The first new electromagnetic four-vector is

$$(J \cdot E, E \times B) \tag{16}$$

with the corresponding extremization condition

$$J \cdot E \;=\; |E \times B| \tag{17}$$

This relation establishes a proportionality between internal dissipation J·E and the magnitude of the Poynting flux $|E \times B|$ [4,6]. Within an electromagnetic flux tube, J represents the internal axial current density and E the axial electric field. The quantity $J \cdot E$ therefore represents energy dissipation per unit volume per unit time, while $E \times B$ is related to the directional energy flux through the Poynting vector. The Poynting vector is defined as

$$S \;=\; \left(\frac{1}{\mu}\right)(E \times B) \tag{18}$$

and represents the directional energy flux density of an electromagnetic field [5,7]. The extremization condition (17) thus suggests a fundamental relationship between energy dissipated within a flux tube and energy transported along it.

The second new electromagnetic four-vector is

$$(\hbar\omega, v \times B) \tag{19}$$

with extremization condition

$$\hbar\omega \;=\; |v \times B| \tag{20}$$

Here $\hbar\omega$ represents the quantum energy of a photon or fermionic mode, while $v \times B$ appears in the Lorentz force expression [1,9]. This four-vector connects quantum mechanical energy with electromagnetic field quantities. For a fermion propagating along an electromagnetic helix, the velocity v satisfies $v < c$ for massive particles, while for photons and massless gauge bosons, $v = c$ [11]. The extremization condition (20) therefore provides a bridge between particle energy and field-mediated forces. A solution to this four-vector construction exists in the form of a massive helix. According to the constant-mass acceleration mechanism (previously termed "Reverse Higgs"), we obtain

$$\Delta(\hbar\omega) \;=\; mv^2 \tag{21}$$

where m remains constant throughout the acceleration process [6,13]. It then follows immediately that extremization of this new four-vector is satisfied. For circular motion corresponding to centripetal force, we have the relation

$$\frac{mv^2}{R} \leftrightarrow \hbar\omega \tag{22}$$

where R is the radius of curvature [2]. This correspondence suggests that the energy quanta ħω can be associated with the kinetic energy of circular motion. The standard relation

$$\hbar\nu \;=\; \hbar\omega \tag{23}$$

may be used interchangeably depending on context, with ν representing frequency in hertz and ω representing angular frequency [9]. This four-vector analysis yields the physical result that if a massive fermion propagates along an electromagnetic helix and is rendered massless via the constant-mass acceleration mechanism, it continues to propagate on that same helix. Prior to establishing this four-vector, this outcome was conjectural [1].

The two processes between which we oscillate are Einsteinian spacetime curvature and Newtonian mechanics. Within this framework, the complex conjugate of the Poynting vector may be considered,

$$(J \cdot E, E *\!\times B) \tag{24}$$

This permits the concept of equal net helical and axial currents. This result is consistent with the limiting relations

$$E \;\to\; 0, v \;\to\; c \tag{25}$$

and

$$E \;\to\; \infty, v \;\to\; 0 \tag{26}$$

These limits describe the transition between radiative and static regimes [4,5]. We develop what may be termed a "Stokes' strip analysis," revealing that the magnetic analogue of the electric field near a charged plane depends solely on charge density. For a uniformly charged plane, the electric field is

$$E \;= \frac{\sigma}{2\varepsilon^0} \tag{27}$$

where σ is the surface charge density [5]. In the magnetic case of an electromagnetic flux tube, the component of the helical current that contributes is the azimuthal component.

Consequently, the Poynting flux at the surface equals the internal Poynting flux, even though one is a linear flux density and the other an area flux density; moreover, they are orthogonal. This can be expressed as

$$|S_{surface}| = |S_{internal}| \quad (28)$$

with the directions satisfying $S_{surface} \cdot S_{internal} = 0$. Finally, the internal axial fields, related by $J \leftrightarrow E$, become determined by the coiling density of the helical fields, forming the magnetic analogue of the electric field near a surface of fixed charge density. The coiling density n (turns per unit length) relates to the pitch angle α by

$$n = \frac{\tan\alpha}{2\pi R} \quad (29)$$

where R is the flux tube radius [10,12]. The axial magnetic field B_{ax} is then related to the azimuthal field B_θ by

$$B_\theta = B_{ax} \tan\alpha \quad (30)$$

These geometric relations determine the internal field configuration and its associated energy transport properties. The two four-vectors introduced in this section $(J \cdot E, E \times B)$ and $(\hbar\omega, v \times B)$ together with their extremization conditions, form the basis for the theoretical developments that follow. They represent a unification of dissipation, energy transport, quantum energy, and force-mediated dynamics within a single phenomenological framework [1,6].

THEORETICAL IMPLICATIONS AND FORMAL RESULTS

A fundamental force is normally expected to possess a Lagrangian formulation. Since no such Lagrangian has been firmly established for gravity in a manner consistent with quantum mechanics, it is commonly assumed that gravity is not a force but rather a manifestation of spacetime curvature [3,7]. Yet gravitational interactions between atomic nuclei are demonstrably forces, not curvature effects at nuclear scales. Musakhail's theory is formulated entirely as a force theory, and it explicitly includes gravity, insofar as the motion between the Earth's surface and nominally stationary space just outside that surface defines an aether velocity and therefore an aether force [1]. Accordingly, we establish below an aether Lagrangian, which appears to be a formulation consistent with gravitational interactions at nuclear scales. Musakhail's work on gravitational aether dynamics was developed through an extensive investigation of the Earth's gravitational potential, whose gradient defines the gravitational force [1,6]. It is a perspective in modern physics that this process requires spacetime curvature; indeed, such curvature interpretations may complicate otherwise precise calculations at nuclear scales. We now introduce the Lagrangian and Hamiltonian formalisms. Each underpins an entire mathematical framework: Lagrangian dynamics and Hamiltonian dynamics [2]. Let L denote the Lagrangian and H the Hamiltonian, with T representing kinetic energy and V representing potential energy. In the context of special relativity, we may identify the rest energy contribution as

$$V = m_0 c^2 \quad (31)$$

Thus, the Lagrangian and Hamiltonian take the forms

$$L = T - V \quad (32)$$

$$H = T + V \quad (33)$$

This raises an important conceptual question: why is it useful to consider simultaneously both the difference and the sum of T and V? The present analysis aims to clarify this point through the correspondence between Einstein's special relativity and Musakhail's aether dynamics [1,3]. We therefore confront two distinct theoretical frameworks: Einstein's special relativity, fundamentally an energy-based theory, and Musakhail's aether dynamics, formulated as a force-based theory. The relativistic energy expression is

$$E = m_0 c^2 + \left(\frac{1}{2}\right) m v^2 \quad (34)$$

to first order, where the relativistic mass is given by

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (35)$$

Einstein's energy theory is naturally associated with a Hamiltonian, $H = T + V$, whereas Musakhail's force theory is described by a Lagrangian, $L = T - V$ [1,2]. This distinction is precisely what we would expect from a force formulation, since the other three fundamental interactions the electromagnetic, weak nuclear, and strong nuclear forces are all described by Lagrangians [5,9].

By contrast, beyond Newton's original formulation, remarkably little has been established regarding gravitational force, to the extent that many physicists now decline to regard any theory of gravity involving "force" as worthy of consideration. In the present discussion, we examine whether gravitational interactions involving nuclei rather than "bare fermions" such as photons or massive e^+/e^- constitute a force-based description. Consequently, such a description would possess a Lagrangian, and that Lagrangian would take the form $L = T - V$ [1,6]. The aether force is given by

$$F = c^2(m - m_0) \quad (36)$$

which serves as the basis for the gravitational Lagrangian [1]. For $v > c$, the mass m becomes imaginary, a regime that may correspond to particular physical conditions requiring further investigation. We now seek to establish fundamental connections between Lagrangians and Hamiltonians, aether forces, and Einsteinian special-relativistic energy analysis. The distinction between force and energy cannot be overemphasized. Accordingly, we proceed by examining the two new electromagnetic four-vectors obtained in our investigations. These four-vectors, together with the results of their extremization, are given by

$$(J \cdot E, E \times B) \rightarrow J \cdot E = |E \times B| \qquad (37)$$

and

$$(\hbar\omega, v \times B) \rightarrow \hbar\omega = |v \times B| \qquad (38)$$

We begin with the second relation, $\hbar\omega = |v \times B|$. Here, the extremization of this electromagnetic four-vector implies that energy is equivalent to force in a certain sense, demonstrating that the Lagrangian and Hamiltonian emerge simultaneously from the dynamics of the variables T and V. Recall that the Lorentz force is given by

$$F = q(E + v \times B) \qquad (39)$$

In the present context, we are concerned specifically with the vector product term. Note that the charge factor q does not appear in the four-vector expression (38); there are reasons for this, discussed in earlier investigations. This is likely related to the fact that q plays no role when the Lorentz force is nullified via $E = -v \times B$. We now turn to the other four-vector uncovered in our investigations: $(J \cdot E, E \times B)$. This involves electrical dissipation $J \cdot E$, representing energy dissipated per unit time per unit volume. On the right-hand side, one has the irradiated energy $|E \times B|$, again expressed per unit time per unit area. This was the original motivation for constructing this four-vector. The quantity $|E \times B|$ is taken directly from the Poynting vector,

$$P = \left(\frac{1}{\mu}\right)(E \times B) \qquad (40)$$

Note again that the factor $1/\mu$ does not appear in the four-vector (37), just as previously the charge factor $q = 1.6 \times 10^{-19}\,C$ might have been expected to appear. The motivation for excluding these factors was guided by mathematical and physical simplicity; moreover, extensive investigation suggests that such simplicity often provides reliable guidance in formulating physical theory. We now consider the scalar term in the four-vector $(J \cdot E, E \times B)$. Here, $J \cdot E$ represents the rate of dissipation. In classical physics, heat dissipation is simply power, P, defined as heat dissipated per unit time and given by force multiplied by velocity. In an electromagnetic circuit, the force is qE; however, consistent with the above four-vector, we omit the factor q, and the electric field is taken to be the internal axial field of the circuit. The axial velocity in the interior of the flux tube is therefore c. Thus, just as for the other electromagnetic four-vector introduced in these discussions, extremization leads directly to a statement linking energy dissipation and energy propagation. We therefore obtain equation (37) as before. The magnetic field can then be eliminated using the transformation

$$E = cB \qquad (41)$$

This relation arises from earlier results and represents the connection between the internal axial currents of a flux tube the field lines E_{ax} and B_{ax} along which fermions (here termed "weak-strong gauge bosons") propagate in opposite directions. It follows from negation of the Lorentz force, occurring primarily so that photons, which are electrically neutral, do not interact with external electromagnetic fields in the same manner as charged particles. Starting from $F = q(E + v \times B) = 0$, we obtain

$$E = -v \times B \qquad (42)$$

For a photon, $v = c$, yielding $E = cB$ in magnitude. It is straightforward to verify that the Poynting vector P, discussed above, which points in the direction of v, satisfies the directional properties implied by this vector cross product [5,7]. The simplicity of the analysis specifically the removal of q from consideration provides further support for excluding q from the relevant four-vector. What was initially motivated by aesthetic considerations finds justification in mathematical consistency. We therefore proceed with the analysis by eliminating B. From $|E|c = |E||B|$, we obtain

$$|E| = \mu c^2 \qquad (43)$$

Here, E serves not only as a propagation vector for fermions, positrons, and positive electric charge, but also as an electromagnetic impulse. As with B, in the electromagnetic amplitude domain E is orthogonal to B, rather than oppositely directed as in the propagation-vector scenario. Similarly, electromagnetic and quantum theory introduce a third amplitude, m, appearing in the de Broglie relation

$$mv\lambda = h \qquad (44)$$

Since the de Broglie relation coexists with Planck's law,

$$E = h\nu \qquad (45)$$

we conclude that m may represent either the amplitude of a wave with v < c in a Bohr orbit or the amplitude of an electromagnetic wave with $v = c$. Thus, transitioning from the Bohr picture to the Schrödinger regime equivalently, to the dual Maxwellian radiation scenario we identify

$$m \leftrightarrow |E|, |B| \qquad (46)$$

Which should be chosen, $|E|$ or $|B|$? There is no compelling reason to favor B over E; moreover, internally to a circuit, E aligns with conventional current, corresponding to the passage of positive electric charge. Accordingly, we take $|E| \leftrightarrow m$ as the relevant amplitude. This identification is reinforced by mass-energy equivalence: E is simultaneously an internal axial energy along the flux tube, and therefore m represents energy as well as amplitude. This leads directly to

$$E = m\mu c^2 \qquad (47)$$

This result is in complete analogy with Musakhail's aether force expression (36). Both aspects of physical reality aether force dynamics and Einsteinian energy analysis share the common factor c^2, precisely as expected. Physical constants such as c should naturally appear when considering the fundamental structure $H/L = T \pm V$. The Lagrangian underlying this force-based description of gravitational interactions is therefore

$$L = c^2(m - m_0) \tag{48}$$

When $v > c$, m becomes imaginary, a regime that may correspond to particular gravitational conditions requiring further investigation. Replacing rest mass m_0 with a characteristic mass m_c and allowing v to scale with gravitational interaction distance leads naturally to a gravitational Lagrangian applicable at nuclear scales.

Thus, electromagnetic interaction and gravity share an analogous Lagrangian structure. This suggests why quantum-gravity exchange between nuclei might appear as electric gravitons in one direction and magnetic gravitons in the other. Finally, the characteristic mass m_c corresponds to gravitational mass extrapolated to the minimal gravitational interaction distance two adjacent protons separated by $2R$, where R is the proton radius. At this scale, $v \to c$ and an electro-gravitational correspondence emerges: both interactions share $v = c$, but electromagnetic mass is real and positive, while gravitational mass may require different interpretation. The constants m_0 and m_c are distinct. Electrodynamics concerns photon mass (zero in standard theory), gravitation concerns graviton mass, and in general relativity the two are unified through spacetime geometry.

TESTABLE CONSEQUENCES AND PHYSICAL IMPLICATIONS

Although the present framework is phenomenological and exploratory, it leads to several qualitative consequences that may be examined experimentally or observationally. These predictions, while not yet developed to quantitative precision, offer potential avenues for empirical assessment and theoretical refinement.

Electromagnetic Flux-Tube Energy Transport

The extremization of the four-vector $(J \cdot E, E \times B)$ implies a direct proportionality between internal dissipation and Poynting flux, suggesting that energy transport in helical electromagnetic flux-tube structures should obey the scaling relation

$$|J \cdot E| \propto |E \times B| \tag{49}$$

This prediction may be examined in laboratory-scale electromagnetic helices or plasma flux tubes by measuring axial current density and transverse field components. For a cylindrical flux tube of radius R, the total power dissipated per unit length is

$$P_{diss} = \int (J \cdot E) dA \approx J \cdot E \cdot \pi R^2 \tag{50}$$

while the total power transported along the tube is

$$P_{trans} = \int |S| dA \approx \frac{|E \times B|}{\mu} \cdot \pi R^2 \tag{51}$$

where S is the Poynting vector. Deviations from standard magnetohydrodynamic expectations would provide a direct test of the proposed phenomenological model. In laboratory plasmas, helical flux tubes can be generated and diagnosed using magnetic probes and interferometry. The scaling relation (49) predicts that the ratio

$$\eta = \frac{(J \cdot E)}{|E \times B|} \tag{52}$$

should remain approximately constant under varying experimental conditions, independent of plasma parameters such as density and temperature. This constancy would indicate a fundamental relationship between dissipation and energy transport beyond conventional plasma physics.

Fermionic Helices and Constant-Mass Acceleration

The extremization of $(\hbar\omega, v \times B)$ together with the constant-mass acceleration mechanism introduced here suggests that fermions propagating along electromagnetic helices undergo acceleration to relativistic velocities while maintaining approximately constant total mass [9]. This behavior may be expressed as

$$\Delta(\hbar\omega) = mv^2 \tag{53}$$

with m constant throughout the acceleration process. This phenomenon may be investigated using high-energy electron beams or synchrotron systems, where longitudinal acceleration and transverse magnetic confinement coexist. In a helical undulator or wiggler, electrons follow sinusoidal trajectories and emit radiation at characteristic frequencies [5]. The constant-mass acceleration mechanism predicts specific correlations between the electron energy gain and the emitted radiation spectrum. For an electron in a helical magnetic field B with pitch angle α, the transverse velocity component is

$$v_\perp = v \sin\alpha \tag{54}$$

and the longitudinal component is

$$v_\parallel = v \cos\alpha \tag{55}$$

The emitted radiation frequency in the forward direction is given by

$$\omega = \frac{2\gamma^2 \omega_0}{1 + \frac{K^2}{2}} \tag{56}$$

where ω_0 is the undulator period frequency and K is the undulator parameter. The constant-mass acceleration mechanism would manifest as deviations from the standard energy-frequency relation under specific conditions.

Observable signatures would include characteristic correlations between emitted radiation spectra and applied helical field geometries.

If the fermion mass remains constant during acceleration, the usual relativistic mass increase with velocity would be suppressed, leading to measurable differences in radiation patterns [1,11].

Gravitational Analogy and Finite Interaction Time

Within the force-based gravitational interpretation, planetary-scale gravitation is modeled as collective nuclear interactions mediated by graviton exchange with finite propagation time. The framework predicts small phase shifts in orbital dynamics arising from this finite interaction delay, analogous to rotational corrections in electromagnetic helices.

For a two-body gravitational system, the finite propagation time τ of gravitational interaction leads to term in the equation of motion. If the interaction propagates at speed c over distance r, the time delay is

$$\tau = \frac{r}{c} \tag{57}$$

This delay introduces to the Newtonian potential, which to first order may be expressed as

$$V_{eff(r)} = -\left(\frac{GMm}{r}\right)\left[1 + \left(\frac{v^2}{c^2}\right) + \ldots\right] \tag{58}$$

where v is the orbital velocity [8]. Such corrections are analogous to those appearing in post-Newtonian approximations to general relativity. For planetary orbits, this finite interaction time produces a secular precession of the perihelion. The precession angle per orbit is approximately

$$\Delta\varphi \approx \frac{6\pi GM}{c^2 a(1-e^2)} \tag{59}$$

where a is the semi-major axis and e is the eccentricity. This expression matches the well-known general relativistic result for Mercury, suggesting that force-based interpretations with finite interaction time may reproduce observational data.

Such effects may be sought in high-eccentricity orbital systems or precision timing measurements, providing a potential observational avenue for distinguishing force-mediated gravitational dynamics from purely geometric formulations. Binary pulsar systems offer particularly promising laboratories, as their orbital decay and precession can be measured with high precision.

Outlook for Experimental Assessment

While the present work does not provide quantitative predictions with precise numerical coefficients, these qualitative consequences motivate future efforts toward constructing simplified dynamical models and identifying measurable observables. Laboratory electromagnetic helices and astrophysical orbital systems represent promising platforms for assessing the viability of the proposed phenomenological correspondence. Specific experimental avenues include:

1. **Laboratory plasma experiments**: Measurement of $J \cdot E$ and $E \times B$ in controlled flux-tube configurations using magnetic probes, Langmuir probes, and interferometry.
2. **Particle accelerator studies**: Investigation of electron beam dynamics in helical undulators and wigglers, with precise measurement of energy gain and radiation spectra.
3. **Astrophysical observations**: Precision timing of binary pulsars and analysis of planetary perihelion precession using radar ranging and spacecraft tracking.
4. **Gravitational wave detection**: Possible signatures in gravitational wave signals from compact binary in spirals that deviate from general relativistic predictions.

The constant-mass acceleration mechanism, in particular, could be tested in next-generation laser-plasma accelerators, where electrons are accelerated to GeV energies over centimeter scales. If the electron mass remains constant during acceleration, the usual limitations imposed by relativistic mass increase would be circumvented, leading to higher achievable energies than currently predicted.

SCOPE AND LIMITATIONS

The framework presented in this work is intended as a phenomenological and exploratory model rather than a complete field-theoretic formulation. No fundamental action or covariant field equations are derived from first principles, and the four-vector constructions are introduced heuristically to examine internal consistency and physical analogies.

Mathematical Formalism

The extremization procedure employed throughout this study setting the scalar component of a four-vector equal to the magnitude of its vector component is applied heuristically and does not arise from a variational principle applied to an action. Specifically, for a general four-vector (S, V), we impose

$$S = |V| \tag{60}$$

without deriving this condition from an action of the form

$$\delta \int L\, d^4x = 0 \tag{61}$$

where L would be a Lagrangian density. This limits the status of the resulting relations to phenomenological observations rather than fundamental laws. The four-vectors introduced, as equations (6), (7), (8) and (11) are not shown to transform covariantly under Lorentz transformations in the strict sense, although they are constructed by analogy with known four-vectors. A proper four-vector must satisfy

$$V'^{\mu} = \Lambda^{\mu}_{\nu} V^{\nu} \tag{62}$$

under Lorentz transformations, where Λ^{μ}_{ν} is the Lorentz transformation matrix. This property has not been rigorously demonstrated for the newly proposed four-vectors.

Gravitational Interpretation

The gravitational interpretation proposed here treats interactions between atomic nuclei as force-mediated processes via graviton exchange. However, quantitative predictions for macroscopic systems are not developed. The relation between nuclear-scale forces and planetary-scale gravitation remains qualitative:

$$F_{gravity(planetary)} \approx \Sigma F_{nuclear} \quad (63)$$

where the summation over all nuclei is not explicitly defined mathematically. The framework suggests that graviton velocity scales with interaction distance,

$$v_g \propto r \quad (64)$$

yielding constant interaction time

$$\tau = \frac{r}{v_g} = constant \quad (65)$$

but this scaling relation is not derived from underlying dynamics and lacks empirical justification.

Electromagnetic Helices and Constant-Mass Acceleration

The electromagnetic helices and constant-mass acceleration mechanism (previously termed "Reverse Higgs") are discussed qualitatively, and their detailed dynamics remain to be established through explicit modeling. The constant-mass condition

$$m = constant\ during\ acceleration \quad (66)$$

contradicts the standard relativistic relation

$$m = \gamma m_0 \quad (67)$$

with $\gamma = 1/\sqrt{\left(1 - \frac{v^2}{c^2}\right)}$. Reconciliation with special relativity would require either a modification of relativistic dynamics or a restricted regime where the effect applies. The relation

$$\Delta(\hbar\omega) = mv^2 \quad (21)$$

from Section 3 assumes constant mass but does not specify how this reconciles with energy conservation in relativistic contexts.

Four-Vector Extremization

The extremization conditions

$$J \cdot E = |E \times B| \quad (17)$$

and

$$\hbar\omega = |v \times B| \quad (20)$$

are postulated rather than derived. While they reveal interesting structural analogies, their physical status remains uncertain. Dimensional analysis reveals potential inconsistencies: $J \cdot E$ has units of power density (W/m³), while $|E \times B|$ has units of intensity (W/m²) when divided by μ [4,5]. The equality (17) therefore requires an implicit length scale ℓ such that

$$J \cdot E = \left(\frac{1}{\ell}\right) |E \times B| \quad (68)$$

This length scale has not been identified or derived from the theory. Similarly, $\hbar\omega$ has units of energy (J), while $|v \times B|$ has units of force per unit charge times velocity (N·m/C·s^2), requiring careful dimensional reconciliation.

Empirical Status

The testable consequences outlined in Section 5 remain qualitative. Specific numerical predictions are not provided, and experimental signatures are not quantified. For example, the perihelion precession expression

$$\Delta\varphi \approx \frac{6\pi GM}{c^2 a(1 - e^2)} \quad (59)$$

matches the general relativistic result but is presented as an approximation without derivation from the force-based framework. The flux-tube energy transport scaling

$$|J \cdot E| \propto |E \times B| \quad (49)$$

lacks a predicted proportionality constant that could be compared with experiment.

Relationship to Established Theories

The framework does not replace or generalize established theories such as general relativity or quantum electrodynamics. Rather, it explores conceptual correspondences that may suggest new lines of inquiry. Key differences from standard theory include:

1. **Gravity as force**: Standard general relativity treats gravity as spacetime curvature, not as a force in the Newtonian sense.
2. **Aether dynamics**: The concept of an aether is not part of modern relativistic physics, though it appears in some alternative approaches.
3. **Constant-mass acceleration**: This contradicts special relativity unless interpreted within a restricted domain.
4. **Four-vector extremization**: Not a standard technique in field theory.

Outlook for Further Development

Accordingly, the present results should be viewed as conceptual rather than definitive. Further mathematical development, numerical simulations, and targeted experiments are required to assess the physical validity and predictive power of the proposed correspondence between electromagnetic and gravitational dynamics. Specific areas requiring development include:

1. **Covariant formulation**: Deriving the four-vectors from an action principle to ensure Lorentz covariance.
2. **Quantitative predictions**: Calculating numerical coefficients for experimental signatures.
3. **Reconciliation with relativity**: Clarifying the relationship between constant-mass acceleration and special relativity.
4. **Microscopic foundation**: Deriving nuclear-scale force laws from more fundamental principles.
5. **Experimental design**: Proposing specific experiments with measurable outcomes.

The framework is presented in the spirit of exploratory science, acknowledging its limitations while suggesting potential avenues for future investigation.

DISCUSSION

The two new electromagnetic four-vectors introduced in this work $(J \cdot E, E \times B)$ and $(\hbar\omega, v \times B)$ appear to be mutually consistent through their extremization conditions. The question then arises whether they are also consistent with other established four-vectors, particularly the electromagnetic four-potential (φ, A) and the spacetime four-vector (ct, x). For comparison, we consider the four-vector (φ, A), where the scalar component φ is the electric potential and the vector component A is the magnetic vector potential. Extremization of this four-vector yields

$$\varphi = |A| \tag{69}$$

This relation, while not standard in electromagnetic theory, provides a heuristic connection between the scalar and vector potentials under specific conditions. We next recall key properties of the magnetic vector potential A, equation (9) and together with its complementary relation from Ampère's law (equation 10).

Consider an electromagnetic flux tube: internally there is an axial current J, while at the surface there exists a helical magnetic field B, which acts to confine and stabilize the flux tube.

Extending this picture to solar flares representing enormously macroscopic electromagnetic flux tubes we observe an analogous structure. In this case, the helical surface field lines are no longer effectively one-dimensional, as in terrestrial electromagnetic flux tubes, but instead become macroscopic magnetic structures of substantial thickness. This macroscopic magnetic field B is itself confined by a helical vector potential A at the surface, in direct analogy with the helical B field that confines the J flux tube at smaller scales.

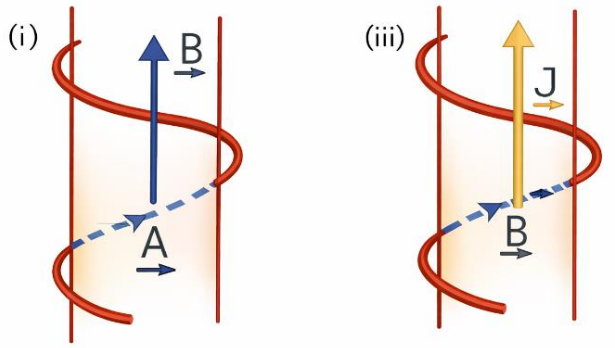

Figure 1. (i) Macroscopic helical magnetic field B of a solar flare, confined by a curling vector potential A at the flux-tube surface. (ii) Analogous electromagnetic circuit showing axial current density J and helical surface field B, illustrating the structural equivalence between solar flux tubes and laboratory circuits.

We now relate the electric field strength E to the electric potential φ via

$$E = -\nabla\varphi \tag{70}$$

For a static potential, this gives the electrostatic field. Through extremization condition (69), we have $\varphi = |A|$, and therefore

$$E = -\nabla|A| \tag{71}$$

From Ampère's law (10) we then obtain

$$\mu J = \nabla \times B \tag{10}$$

Taking the dot product with $\nabla|A|$ and using (71) yields

$$\mu(J \cdot E) = (\nabla \times B) \cdot \nabla|A| \tag{72}$$

This result is particularly significant as it connects the dissipation term $J \cdot E$ with the magnetic field configuration. We now invoke the symmetry $B \leftrightarrow A$, that is, we interchange A and B. First, this provides an appealing symmetry, and we continually seek such structures in physical theory. Second, imposing this symmetry allows the above vector expression to simplify naturally, with physical insight as the anticipated outcome. We therefore write

$$\mu(J \cdot E) = (\nabla \times A) \cdot \nabla|B| = B \cdot \nabla|B| \tag{73}$$

Expanding, this becomes

$$B \cdot \nabla|B| = B_x\left(\frac{\partial|B|}{\partial x}\right) + B_y\left(\frac{\partial|B|}{\partial y}\right) + B_z\left(\frac{\partial|B|}{\partial z}\right) \tag{74}$$

We are concerned specifically with axial propagation inside an electromagnetic flux tube, corresponding to the x-direction (unit vector i), and we neglect propagation in the transverse directions. For an axial field B_x, it is unclear how $(\partial B\_x)/\partial x$ behaves, since B_x may vary downstream in conjunction with an opposing variation of the internal axial electric field E_x, which points opposite to B_x. Consequently, rather than differentiating a propagation vector, we instead differentiate an amplitude. This distinction is essential: electromagnetic vectors appear both as wave amplitudes and as propagation vectors for fermion-carrying waves, and the two are intimately connected. Accordingly, we take an amplitude of the form

$$B_x = B_0 \exp[j(kx - \omega t)] \tag{75}$$

where $j = \sqrt{-1}$. Differentiation gives

$$\frac{\partial B_x}{\partial x} = jk\, B_x = (jk)B_x \tag{76}$$

Using the dispersion relation for electromagnetic waves, $\omega = ck$, and the relation between electric and magnetic fields, $E_x = cB_x$, we obtain

$$jk = j\left(\frac{\omega}{c}\right) = j\left(\frac{\omega}{E_x}\right)B_x \tag{77}$$

More systematically, we note that

$$\frac{\partial B_x}{\partial x} = \left(\frac{k}{\omega}\right)\frac{\partial B_x}{\partial t} \tag{78}$$

The factor j corresponds to a 90° rotation in the complex plane. Likewise, because electromagnetic waves consist of two orthogonal oscillations E and B, this rotation maps $E \leftrightarrow B$. Thus, the magnetic component of an electromagnetic wave is inherently imaginary in a formal sense, although this point requires careful interpretation [1,3]. We therefore obtain

$$\mu(J \cdot E) = B_x \left(\frac{\partial B_x}{\partial x}\right) i = \left(\frac{k}{\omega}\right) E_x B_x = \left(\frac{p}{\hbar}\right) E \times B \quad (79)$$

where we have invoked the de Broglie relation

$$p = mv = \frac{h}{\lambda} = \hbar k \quad (80)$$

noting that for the constant-mass acceleration process, m remains constant. At this stage we are not concerned with precise numerical constants in these four-vector calculations only that they remain constant within the heuristic framework. This derivation shows that the extremization conditions and four-vector constructions exhibit internal consistency when examined through the lens of flux-tube geometry and wave propagation. The appearance of the de Broglie relation links the classical field quantities with quantum mechanical momentum, suggesting a deeper connection between the two domains.

The structural analogy between solar flare flux tubes and laboratory circuits illustrated in Figure 1 highlights the self-similar nature of these configurations across dramatically different scales. In both cases, a helical field confines an axial current, and the resulting topology determines the energy transport and dissipation properties.

This scale invariance suggests that the underlying dynamics may be governed by similar principles whether the system is meters or thousands of kilometers in extent. Such self-similarity is characteristic of systems described by scale-invariant field equations and may indicate that the proposed four-vector constructions capture essential features of electromagnetic flux tubes across all scales.

The correspondence between (φ, A) extremization and the flux-tube confinement geometry further supports the heuristic value of the extremization principle. While not derived from first principles, it reveals structural parallels that might otherwise remain hidden.

CONCLUSION

At its core, this paper concerns the duality between Einstein's special relativity and Musakhail's aether dynamics, interpreted through the lens of Lagrangian-Hamiltonian correspondence. Our investigations were carried out in connection with what may be termed the relation between energy-based and force-based descriptions of fundamental interactions.

The relativistic energy expression (1) under Taylor expansion for low velocities reduces to the approximate form (34). While mathematically convenient for low velocities, this approximation may obscure essential physical information specifically, the constant-mass acceleration mechanism examined in this work. Because constant-mass acceleration involves velocity ranging from zero to the speed of light, it cannot be fully captured by an expansion about low velocities. The resulting approximation implicitly assumes constant mass, which is precisely the condition examined in the constant-mass acceleration process.

In the Musakhail-Einstein correspondence, we therefore require Einstein's special relativity in its full form. Any analysis restricted to low velocities may obscure critical physics. Initially, electric and magnetic fields appear axially and oppositely directed within a flux tube; however, upon transforming to the amplitude representation, the fields become orthogonal, yielding the transition from axial fields to the cross product that appears in the Poynting vector. In this way, we confirm the electromagnetic four-vector (8) through examination of the potential four-vector (6), provided the proportionality factors remain constant.

We therefore consider the regime where velocity equals the speed of light with mass constant, naturally identifying the mass with the electron mass in appropriate contexts. This assumption is physically consistent: in the constant-mass acceleration mechanism, an electron accelerated from rest to the speed of light maintains constant total mass throughout the process, as described by relation (35). The limiting behavior as velocity approaches the speed of light requires the rest mass to approach zero while preserving the full relativistic energy-momentum relation (1).

Einstein's Taylor expansion discards this regime, and subsequent treatments may have followed suit. The constant-mass acceleration mechanism restores Newtonian dynamics in the sense that total mass, acceleration, and force remain constant during the process. Since Musakhail's theory is fundamentally a force theory, it participates centrally in constant-mass acceleration dynamics. Only under constant force can the mass-energy equivalence be interpreted within a generalized kinematics.

With constant mass and constant acceleration, elementary kinematics yields relation (81). For the constant-mass acceleration case with velocity reaching the speed of light, this gives relation (82). The photon energy can then be expressed as in (83), establishing photon energy directly from force-based dynamics under the constant-mass acceleration assumption. Our final observation is that Musakhail's theory is fundamentally a theory of force, and because it incorporates gravity, gravity may be consistently interpreted as a force within the present phenomenological framework. Musakhail introduced gravity via processes inside black holes and through planetary gravitational fields. The Musakhail Lagrangian underlying this description takes the form (48). When velocity exceeds the speed of light, mass becomes imaginary, a regime that may correspond to particular gravitational conditions requiring further investigation. Replacing rest mass with a characteristic mass and allowing velocity to scale with gravitational interaction distance leads naturally to a gravitational Lagrangian applicable at nuclear scales. Thus, electromagnetic interaction as described by Einsteinian special relativity and gravity share an analogous Lagrangian structure. This suggests why quantum-gravity exchange between nuclei might appear as electric gravitons in one direction and magnetic gravitons in the other. Finally, the characteristic mass corresponds to gravitational mass extrapolated to the minimal gravitational interaction distance two adjacent protons separated by twice the proton radius. At this scale, velocity approaches the speed of light and an electro-gravitational correspondence emerges: both interactions share the same characteristic speed, but electromagnetic mass is real and positive, while gravitational mass may require different interpretation. The rest mass and the characteristic mass are distinct quantities. Electrodynamics concerns photon mass, gravitation concerns graviton mass, and in general relativity the two are unified through spacetime geometry.

The exploratory framework presented here does not replace established theories but rather suggests conceptual correspondences that may motivate further theoretical development and experimental investigation. The two new four-vectors (8) and (11), the extremization principle (17) and (20), and the constant-mass acceleration mechanism (21) offer heuristic tools for exploring possible connections between electromagnetic and gravitational phenomena.

Future work should focus on developing a covariant formulation of the proposed four-vectors, deriving quantitative predictions for experimental tests, reconciling the constant-mass acceleration mechanism with special relativity, exploring the nuclear-scale gravitational implications in more detail, and seeking observational evidence for the proposed effects in astrophysical systems.

The speculative nature of this framework is acknowledged, and its physical relevance depends on further mathematical development and empirical validation. In the spirit of exploratory science, these ideas are offered as potential avenues for inquiry rather than established results.

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